

Types of variable normalization formulas

1	Selection of objects and variables	data matrix $[x_{ij}]$		
2	Variable scale level	Ratio	ratio	interval
	Selection of variable normalization formula	n6 – quotient transformation (x/sd) n7 – quotient transformation (x/range) n8 – quotient transformation (x/max) n9 – quotient transformation (x/mean) n10 – quotient transformation (x/sum) n11 – quotient transformation x/sqrt(SSQ)	n1 – standardization n2 – positional standardization n3 – unitization n4 – unitization with zero minimum n5 – normalization in range $[-1, 1]$	n1 – standardization n2 – Weber standardization n3 – unitization n4 – unitization with zero minimum n5 – normalization in range $[-1, 1]$
	Transformed variable scale level	Ratio	interval	interval

$$(n1) \quad z_{ij} = s_j^{-1} x_{ij} - \bar{x}_j s_j^{-1},$$

$$(n2) \quad z_{ij} = (x_{ij} - Me_j) / 1,4826 \cdot MAD_j,$$

$$(n3) \quad z_{ij} = r_j^{-1} x_{ij} - \bar{x}_j r_j^{-1},$$

$$(n4) \quad z_{ij} = \left[x_{ij} - \min_i \{x_{ij}\} \right] / r_j$$

$$(n5) \quad z_{ij} = (x_{ij} - \bar{x}_j) / \max_i |x_{ij} - \bar{x}_j|$$

$$(n6) \quad x_{ij} / s_j$$

$$(n7) \quad x_{ij} / r_j$$

$$(n8) \quad x_{ij} / \max_i \{x_{ij}\}$$

$$(n9) \quad x_{ij} / \bar{x}_j$$

$$(n10) \quad x_{ij} / \sum_{i=1}^n x_{ij}$$

$$(n11) \quad x_{ij} / \sqrt{\sum_{i=1}^n x_{ij}^2}$$

where: $x_{ij} (z_{ij})$ – i -th observation on j -th variable (i -th normalized observation on j -th variable),

\bar{x}_j, s_j – mean and standard deviation for j -th variable,

Me_j, MAD_j – median and median absolute deviation for j -th variable,

$r_j = \max_i \{x_{ij}\} - \min_i \{x_{ij}\}$.

References

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