

Package ‘BivGeo’

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Type Package

Title Basu-Dhar Bivariate Geometric Distribution

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Description Provides functions to compute the joint probability mass function (pmf), cumulative distribution function (cdf), and survival function (sf) of the Basu-Dhar bivariate geometric distribution. Additional functionalities include the calculation of the correlation coefficient, covariance, and cross-factorial moments, as well as the generation of random variates. The package also implements parameter estimation based on the method of moments.

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cfbivgeo

*Cross-factorial Moment for the Basu-Dhar Bivariate Geometric Distribution***Description**

This function computes the cross-factorial moment for the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values.

Usage

```
cfbivgeo(theta)
```

Arguments

theta vector (of length 3) containing values of the parameters θ_1, θ_2 and θ_3 of the Basu-Dhar bivariate Geometric distribution. For real data applications, use the maximum likelihood estimates or Bayesian estimates to get the cross-factorial moment.

Details

The cross-factorial moment between X and Y, assuming the Basu-Dhar bivariate geometric distribution, is given by,

$$E[XY] = \frac{1 - \theta_1\theta_2\theta_3^2}{(1 - \theta_1\theta_3)(1 - \theta_2\theta_3)(1 - \theta_1\theta_2\theta_3)}$$

Note that the cross-factorial moment is always positive.

Value

[cfbivgeo](#) computes the cross-factorial moment for the Basu-Dhar bivariate geometric distribution for arbitrary parameter values.

Invalid arguments will return an error message.

Author(s)

Ricardo P. Oliveira <rpuziol.oliveira@gmail.com>

Jorge Alberto Achcar <achcar@fmrp.usp.br>

Source

[cfbivgeo](#) is calculated directly from the definition.

References

- Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.
- Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, **42**, **2**, 252-266.
- Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.
- de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.
- de Oliveira, R. P., Achcar, J. A., Peralta, D., & Mazucheli, J. (2018). Discrete and continuous bivariate lifetime models in presence of cure rate: a comparative study under Bayesian approach. *Journal of Applied Statistics*, 1-19.

Examples

```
cfbivgeo(theta = c(0.5, 0.5, 0.7))
# [1] 2.517483
cfbivgeo(theta = c(0.2, 0.5, 0.7))
# [1] 1.829303
cfbivgeo(theta = c(0.8, 0.9, 0.1))
# [1] 1.277864
cfbivgeo(theta = c(0.9, 0.9, 0.9))
# [1] 35.15246
```

corbivgeo	<i>Correlation Coefficient for the Basu-Dhar Bivariate Geometric Distribution</i>
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Description

This function computes the correlation coefficient analogous of the Pearson correlation coefficient for the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values.

Usage

```
corbivgeo(theta)
```

Arguments

theta	vector (of length 3) containing values of the parameters θ_1, θ_2 and θ_3 of the Basu-Dhar bivariate Geometric distribution. For real data applications, use the maximum likelihood estimates or Bayesian estimates to get the correlation coefficient.
-------	--

Details

The correlation coefficient between X and Y, assuming the Basu-Dhar bivariate geometric distribution, is given by,

$$\rho = \frac{(1 - \theta_3)(\theta_1\theta_2)^{1/2}}{1 - \theta_1\theta_2\theta_3}$$

Note that the correlation coefficient is always positive which implies that the Basu-Dhar bivariate geometric distribution is useful for bivariate lifetimes with positive correlation.

Value

`corbivgeo` computes the correlation coefficient analogous to the Pearson correlation coefficient for the Basu-Dhar bivariate geometric distribution for arbitrary parameter values.

Invalid arguments will return an error message.

Author(s)

Ricardo P. Oliveira <rpuziol.oliveira@gmail.com>

Jorge Alberto Achcar <achcar@fmrp.usp.br>

Source

`corbivgeo` is calculated directly from the definition.

References

- Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.
- Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, **42**, **2**, 252-266.
- Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu-Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.
- de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.
- de Oliveira, R. P., Achcar, J. A., Peralta, D., & Mazucheli, J. (2018). Discrete and continuous bivariate lifetime models in presence of cure rate: a comparative study under Bayesian approach. *Journal of Applied Statistics*, 1-19.

Examples

```
corbivgeo(theta = c(0.5, 0.5, 0.7))
# [1] 0.1818182
corbivgeo(theta = c(0.2, 0.5, 0.7))
# [1] 0.102009
corbivgeo(theta = c(0.8, 0.9, 0.1))
```

```
# [1] 0.822926
corbivgeo(theta = c(0.9, 0.9, 0.9))
# [1] 0.3321033
```

covbivgeo

*Covariance for the Basu-Dhar Bivariate Geometric Distribution***Description**

This function computes the covariance for the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values.

Usage

```
covbivgeo(theta)
```

Arguments

theta vector (of length 3) containing values of the parameters θ_1, θ_2 and θ_3 of the Basu-Dhar bivariate Geometric distribution. For real data applications, use the maximum likelihood estimates or Bayesian estimates to get the covariance.

Details

The covariance between X and Y, assuming the Basu-Dhar bivariate geometric distribution, is given by,

$$Cov(X, Y) = \frac{\theta_1 \theta_2 \theta_3 (1 - \theta_3)}{(1 - \theta_1 \theta_3)(1 - \theta_2 \theta_3)(1 - \theta_1 \theta_2 \theta_3)}$$

Note that the covariance is always positive.

Value

[covbivgeo](#) computes the covariance for the Basu-Dhar bivariate geometric distribution for arbitrary parameter values.

Invalid arguments will return an error message.

Author(s)

Ricardo P. Oliveira <rpuziol.oliveira@gmail.com>

Jorge Alberto Achcar <achcar@fmrp.usp.br>

Source

[covbivgeo](#) is calculated directly from the definition.

References

- Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.
- Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, **42**, **2**, 252-266.
- Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.
- de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.
- de Oliveira, R. P., Achcar, J. A., Peralta, D., & Mazucheli, J. (2018). Discrete and continuous bivariate lifetime models in presence of cure rate: a comparative study under Bayesian approach. *Journal of Applied Statistics*, 1-19.

Examples

```
covbivgeo(theta = c(0.5, 0.5, 0.7))
# [1] 0.1506186
covbivgeo(theta = c(0.2, 0.5, 0.7))
# [1] 0.04039471
covbivgeo(theta = c(0.8, 0.9, 0.1))
# [1] 0.0834061
covbivgeo(theta = c(0.9, 0.9, 0.9))
# [1] 7.451626
```

dbivgeo

Joint Probability Mass Function for the Basu-Dhar Bivariate Geometric Distribution

Description

This function computes the joint probability mass function of the Basu-Dhar bivariate geometric distribution for arbitrary parameter values.

Usage

```
dbivgeo1(x, y = NULL, theta, log = FALSE)
dbivgeo2(x, y = NULL, theta, log = FALSE)
```

Arguments

x	matrix or vector containing the data. If x is a matrix then it is considered as x the first column and y the second column (y argument need be setted to NULL). Additional columns and y are ignored.
---	---

y	vector containing the data of y. It is used only if x is also a vector. Vectors x and y should be of equal length.
theta	vector (of length 3) containing values of the parameters θ_1, θ_2 and θ_3 of the Basu-Dhar bivariate Geometric distribution. The parameters are restricted to $0 < \theta_i < 1, i = 1, 2$ and $0 < \theta_3 \leq 1$.
log	logical argument for calculating the log probability or the probability function. The default value is FALSE.

Details

The joint probability mass function for a random vector (X, Y) following a Basu-Dhar bivariate geometric distribution could be written in two forms. The first form is described by:

$$P(X = x, Y = y) = \theta_1^{x-1} \theta_2^{y-1} \theta_3^{z_1} - \theta_1^x \theta_2^{y-1} \theta_3^{z_2} - \theta_1^{x-1} \theta_2^y \theta_3^{z_3} + \theta_1^x \theta_2^y \theta_3^{z_4}$$

where $x, y > 0$ are positive integers and $z_1 = \max(x-1, y-1), z_2 = \max(x, y-1), z_3 = \max(x-1, y), z_4 = \max(x, y)$. The second form is given by the conditions:

If $X < Y$, then

$$P(X = x, Y = y) = \theta_1^{x-1} (\theta_2 \theta_3)^{y-1} (1 - \theta_2 \theta_3) (1 - \theta_1)$$

If $X = Y$, then

$$P(X = x, Y = y) = (\theta_1 \theta_2 \theta_3)^{x-1} (1 - \theta_1 \theta_3 - \theta_2 \theta_3 + \theta_1 \theta_2 \theta_3)$$

If $X > Y$, then

$$P(X = x, Y = y) = \theta_2^{y-1} (\theta_1 \theta_3)^{x-1} (1 - \theta_1 \theta_3) (1 - \theta_2)$$

Value

[dbivgeo1](#) gives the values of the probability mass function using the first form of the joint pmf.

[dbivgeo2](#) gives the values of the probability mass function using the second form of the joint pmf.

Invalid arguments will return an error message.

Author(s)

Ricardo P. Oliveira <rpuziol.oliveira@gmail.com>

Jorge Alberto Achcar <achcar@fmrp.usp.br>

Source

[dbivgeo1](#) and [dbivgeo2](#) are calculated directly from the definitions.

References

Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.

Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, **42**, **2**, 252-266.

Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.

de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

See Also

[Geometric](#) for the univariate geometric distribution.

Examples

```
# If x and y are integer numbers:

dbivgeo1(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), log = FALSE)
# [1] 0.16128
dbivgeo2(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), log = FALSE)
# [1] 0.16128

# If x is a matrix:

matr <- matrix(c(1,2,3,5), ncol = 2)

dbivgeo1(x = matr, y = NULL, theta = c(0.2,0.4,0.7), log = FALSE)
# [1] 0.0451584000 0.0007080837
dbivgeo2(x = matr, y = NULL, theta = c(0.2,0.4,0.7), log = FALSE)
# [1] 0.0451584000 0.0007080837

# If log = TRUE:

dbivgeo1(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), log = TRUE)
# [1] -1.824613
dbivgeo2(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), log = TRUE)
# [1] -1.824613
```

mombivgeo

Moments Estimator for the Basu-Dhar Bivariate Geometric Distribution

Description

This function computes the estimators based on the method of the moments for each parameter of the Basu-Dhar bivariate geometric distribution.

Usage

```
mombivgeo(x, y)
```


Arguments

x	matrix or vector containing the data. If x is a matrix then it is considered as x the first column and y the second column (y argument need be setted to NULL). Additional columns and y are ignored.
y	vector containing the data of y. It is used only if x is also a vector. Vectors x and y should be of equal length.

Details

The moments estimators of $\theta_1, \theta_2, \theta_3$ of the Basu-Dhar bivariate geometric distribution are given by:

$$\hat{\theta}_1 = \frac{\bar{Y}(1 - \bar{W})}{\bar{W}(1 - \bar{Y})}$$

$$\hat{\theta}_2 = \frac{\bar{X}(\bar{W} - 1)}{\bar{W}(\bar{X} - 1)}$$

$$\hat{\theta}_3 = \frac{\bar{X}(\bar{X} - 1)(\bar{Y} - 1)}{(\bar{W} - 1)\bar{X}\bar{Y}}$$

Value

[mombivgeo](#) gives the values of the moments estimator.

Invalid arguments will return an error message.

Author(s)

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Source

[mombivgeo](#) is calculated directly from the definition.

References

- Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.
- Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, **42**, **2**, 252-266.
- Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, **9**, 1636-1648.
- de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, **1**, 108-136.

See Also

[Geometric](#) for the univariate geometric distribution.

Examples

```
# Generate the data set:

set.seed(123)
x1 <- rbivgeo1(n = 1000, theta = c(0.5, 0.5, 0.7))
set.seed(123)
x2 <- rbivgeo2(n = 1000, theta = c(0.5, 0.5, 0.7))

# Compute de moment estimator by:

mombivgeo(x = x1, y = NULL) # For data set x1
#           [,1]
# theta1 0.5053127
# theta2 0.5151873
# theta3 0.6640406

mombivgeo(x = x2, y = NULL) # For data set x2
#           [,1]
# theta1 0.4922327
# theta2 0.5001577
# theta3 0.6993893
```

pbivgeo

Joint Cumulative Function for the Basu-Dhar Bivariate Geometric Distribution

Description

This function computes the joint cumulative function of the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values.

Usage

```
pbivgeo(x, y, theta, lower.tail = TRUE)
```

Arguments

x	matrix or vector containing the data. If x is a matrix then it is considered as x the first column and y the second column (y argument need be setted to NULL). Additional columns and y are ignored.
y	vector containing the data of y. It is used only if x is also a vector. Vectors x and y should be of equal length.
theta	vector (of length 3) containing values of the parameters θ_1, θ_2 and θ_3 of the Basu-Dhar bivariate Geometric distribution. The parameters are restricted to $0 < \theta_i < 1, i = 1, 2$ and $0 < \theta_3 \leq 1$.

`lower.tail` logical; If TRUE (default), probabilities are $P(X \leq x, Y \leq y)$ otherwise $P(X > x, Y > y)$.

Details

The joint cumulative function for a random vector (X, Y) following a Basu-Dhar bivariate geometric distribution could be written as:

$$P(X \leq x, Y \leq y) = 1 - (\theta_1 \theta_3)^x - (\theta_2 \theta_3)^y + \theta_1^x \theta_2^y \theta_3^{\max(x, y)}$$

and the joint survival function is given by:

$$P(X > x, Y > y) = \theta_1^x \theta_2^y \theta_3^{\max(x, y)}$$

Value

[pbivgeo](#) gives the values of the cumulative function.

Invalid arguments will return an error message.

Author(s)

Ricardo P. Oliveira <rpuziol.oliveira@gmail.com>

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Source

[pbivgeo](#) is calculated directly from the definition.

References

Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.

Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, **42**, **2**, 252-266.

Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.

de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

See Also

[Geometric](#) for the univariate geometric distribution.

Examples

```
# If x and y are integer numbers:

pbivgeo(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), lower.tail = TRUE)
# [1] 0.79728

# If x is a matrix:

matr <- matrix(c(1,2,3,5), ncol = 2)
pbivgeo(x = matr, y = NULL, theta = c(0.2,0.4,0.7), lower.tail = TRUE)
# [1] 0.8424384 0.9787478

# If lower.tail = FALSE:

pbivgeo(x = 1, y = 2, theta = c(0.2, 0.4, 0.7), lower.tail = FALSE)
# [1] 0.01568

matr <- matrix(c(1,2,3,5), ncol = 2)
pbivgeo(x = matr, y = NULL, theta = c(0.9,0.4,0.7), lower.tail = FALSE)
# [1] 0.01975680 0.00139404
```

rbivgeo

Generates Random Deviates from the Basu-Dhar Bivariate Geometric Distribution

Description

This function generates random values from the Basu-Dhar bivariate geometric distribution assuming arbitrary parameter values.

Usage

```
rbivgeo1(n, theta)
rbivgeo2(n, theta)
```

Arguments

n	number of observations. If $\text{length}(n) > 1$, the length is taken to be the number required.
theta	vector (of length 3) containing values of the parameters θ_1, θ_2 and θ_3 of the Basu-Dhar bivariate Geometric distribution. The parameters are restricted to $0 < \theta_i < 1, i = 1, 2$ and $0 < \theta_3 \leq 1$.

Details

The conditional distribution of X given Y is given by:

If $X < Y$, then

$$P(X = x|Y = y) = \theta_1^{x-1}(1 - \theta_1)$$

If $X = Y$, then

$$P(X = x|Y = y) = \frac{\theta_1^{x-1}(1 - \theta_1\theta_3 - \theta_2\theta_3 + \theta_1\theta_2\theta_3)}{1 - \theta_2\theta_3}$$

If $X > Y$, then

$$P(X = x|Y = y) = \frac{\theta_1^{x-1}\theta_3^{x-y}(1 - \theta_1\theta_3)(1 - \theta_2)}{1 - \theta_2\theta_3}$$

Value

`rbivgeo1` and `rbivgeo2` generate random deviates from the Bash-Dhar bivariate geometric distribution. The length of the result is determined by `n`, and is the maximum of the lengths of the numerical arguments for the other functions.

Invalid arguments will return an error message.

Author(s)

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Source

`rbivgeo1` generates random deviates using the inverse transformation method. Returns a matrix that the first column corresponds to X generated random values and the second column corresponds to Y generated random values.

`rbivgeo2` generates random deviates using the shock model. Returns a matrix that the first column corresponds to X generated random values and the second column corresponds to Y generated random values. See Marshall and Olkin (1967) for more details.

References

- Marshall, A. W., & Olkin, I. (1967). A multivariate exponential distribution. *Journal of the American Statistical Association*, **62**, 317, 30-44.
- Basu, A. P., & Dhar, S. K. (1995). Bivariate geometric distribution. *Journal of Applied Statistical Science*, **2**, 1, 33-44.
- Li, J., & Dhar, S. K. (2013). Modeling with bivariate geometric distributions. *Communications in Statistics-Theory and Methods*, **42**, 2, 252-266.
- Achcar, J. A., Davarzani, N., & Souza, R. M. (2016). Basu–Dhar bivariate geometric distribution in the presence of covariates and censored data: a Bayesian approach. *Journal of Applied Statistics*, **43**, 9, 1636-1648.
- de Oliveira, R. P., & Achcar, J. A. (2018). Basu-Dhar's bivariate geometric distribution in presence of censored data and covariates: some computational aspects. *Electronic Journal of Applied Statistical Analysis*, **11**, 1, 108-136.

See Also

[Geometric](#) for the univariate geometric distribution.

Examples

```
rbivgeo1(n = 10, theta = c(0.5, 0.5, 0.7))
#      [,1] [,2]
# [1,]    2    1
# [2,]    3    1
# [3,]    1    1
# [4,]    1    1
# [5,]    2    2
# [6,]    1    3
# [7,]    2    2
# [8,]    1    1
# [9,]    1    1
# [10,]   2    2
```

```
rbivgeo2(n = 10, theta = c(0.5, 0.5, 0.7))
#      [,1] [,2]
# [1,]    1    1
# [2,]    2    1
# [3,]    2    1
# [4,]    4    1
# [5,]    1    1
# [6,]    2    2
# [7,]    3    2
# [8,]    3    1
# [9,]    3    2
# [10,]   1    1
```

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