

# Package ‘exuber’

July 22, 2025

**Type** Package

**Title** Econometric Analysis of Explosive Time Series

**Version** 1.0.2

**Description** Testing for and dating periods of explosive dynamics (exuberance) in time series using the univariate and panel recursive unit root tests proposed by Phillips et al. (2015) <[doi:10.1111/jere.12132](https://doi.org/10.1111/jere.12132)> and Pavlidis et al. (2016) <[doi:10.1007/s11146-015-9531-2](https://doi.org/10.1007/s11146-015-9531-2)>. The recursive least-squares algorithm utilizes the matrix inversion lemma to avoid matrix inversion which results in significant speed improvements. Simulation of a variety of periodically-collapsing bubble processes. Details can be found in Vasilopoulos et al. (2022) <[doi:10.18637/jss.v103.i10](https://doi.org/10.18637/jss.v103.i10)>.

**License** GPL-3

**URL** <https://github.com/kvasilopoulos/exuber>

**BugReports** <https://github.com/kvasilopoulos/exuber/issues>

**Depends** R (>= 3.2)

**Imports** cli (>= 1.1.0), doRNG (>= 1.8.2), doSNOW (>= 1.0.16), dplyr (>= 1.0.0), foreach (>= 1.4.4), generics (>= 0.0.2), ggplot2 (>= 3.1.1), glue (>= 1.3.1), lubridate (>= 1.7.4), parallel, purrr (>= 0.3.2), Rcpp (>= 0.12.17), rlang (>= 0.3.4), tibble (>= 3.0.2), tidyr (>= 0.8.3), progress (>= 1.2.2)

**Suggests** magrittr (>= 1.5), clisymbols (>= 1.2.0), covr (>= 3.2.1), exuberdata (>= 0.2.0), forcats (>= 0.5.0), gridExtra (>= 2.3), knitr (>= 1.22), rmarkdown (>= 1.12), spelling (>= 2.1), stringr (>= 1.4.0), testthat (>= 2.1.1), withr (>= 2.1.2)

**LinkingTo** Rcpp (>= 1.0.1), RcppArmadillo (>= 0.9.400.2.0)

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|                  |                                  |
|------------------|----------------------------------|
| autoplot.ds_radf | <i>Plotting a ds_radf object</i> |
|------------------|----------------------------------|

---

## Description

Takes a ds\_radf object and returns a ggplot2 object, with a [geom\\_segment\(\)](#) layer.

## Usage

```
## S3 method for class 'ds_radf'  
autoplot(object, trunc = TRUE, ...)
```

## Arguments

|        |                                                                                    |
|--------|------------------------------------------------------------------------------------|
| object | An object of class ds_radf. The output of <a href="#">datestamp()</a>              |
| trunc  | Whether to remove the period of the minimum window from the plot (default = TRUE). |
| ...    | Further arguments passed to methods. Not used.                                     |

## Value

A [ggplot2::ggplot\(\)](#)

## Examples

```
sim_data_wdate %>%  
  radf() %>%  
  datestamp() %>%  
  autoplot()  
  
# Change the colour manually  
sim_data_wdate %>%  
  radf() %>%  
  datestamp() %>%  
  autoplot() +  
  ggplot2::scale_colour_manual(values = rep("black", 4))
```

---

autoplot.radf\_distr      *Plotting a radf\_distr object*

---

### Description

Takes a radf\_distr object and returns a ggplot2 object.

### Usage

```
## S3 method for class 'radf_distr'
autoplot(object, ...)
```

### Arguments

object                      An object of class radf\_distr.  
 ...                          Further arguments passed to methods, used only in wb\_distr facet options.

### Value

A `ggplot2::ggplot()`

---

autoplot.radf\_obj          *Plotting radf models*

---

### Description

autoplot.radf\_obj takes radf\_obj and radf\_cv and returns a faceted ggplot object. shade is used as an input to shape\_opt. shade modifies the geom\_rect layer that demarcates the exuberance periods.

### Usage

```
## S3 method for class 'radf_obj'
autoplot(
  object,
  cv = NULL,
  sig_lvl = 95,
  option = c("gsadf", "sadf"),
  min_duration = 0L,
  select_series = NULL,
  nonrejected = FALSE,
  shade_opt = shade(),
  trunc = TRUE,
  include_negative = "DEPRECATED",
  ...)
```

```

)

## S3 method for class 'radf_obj'
autoplot2(
  object,
  cv = NULL,
  sig_lvl = 95,
  option = c("gsadf", "sadf"),
  min_duration = 0L,
  select_series = NULL,
  nonrejected = FALSE,
  trunc = TRUE,
  shade_opt = shade(),
  ...
)

shade(
  fill = "grey55",
  fill_negative = fill,
  fill_ongoing = NULL,
  opacity = 0.3,
  ...
)

```

### Arguments

|                               |                                                                                                                                                                |
|-------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>object</code>           | An object of class <code>obj</code> .                                                                                                                          |
| <code>cv</code>               | An object of class <code>cv</code> .                                                                                                                           |
| <code>sig_lvl</code>          | Significance level. It could be one of 90, 95 or 99.                                                                                                           |
| <code>option</code>           | Whether to apply the "gsadf" or "sadf" methodology (default = "gsadf").                                                                                        |
| <code>min_duration</code>     | The minimum duration of an explosive period for it to be reported (default = 0).                                                                               |
| <code>select_series</code>    | A vector of column names or numbers specifying the series to be used in plotting. Note that the order of the series does not alter the order used in plotting. |
| <code>nonrejected</code>      | If TRUE, plot all variables regardless of rejecting the NULL at the 5 percent significance level.                                                              |
| <code>shade_opt</code>        | Shading options, typically set using <code>shade</code> function.                                                                                              |
| <code>trunc</code>            | Whether to remove the period of the minimum window from the plot (default = TRUE).                                                                             |
| <code>include_negative</code> | Argument name is deprecated and substituted with <code>nonrejected</code> .                                                                                    |
| <code>...</code>              | Further arguments passed to <code>ggplot2::facet_wrap</code> and <code>ggplot2::geom_rect</code> for <code>shade</code> .                                      |
| <code>fill</code>             | The shade color that indicates the exuberance periods with positive signal                                                                                     |
| <code>fill_negative</code>    | The shade color that indicates the exuberance periods with positive signal                                                                                     |

`fill_ongoing`     The shade color that indicates the exuberance periods that are ongoing the null hypothesis.

`opacity`         The opacity of the shade color aka alpha.

## Value

A `ggplot2::ggplot()`

## Examples

```
rsim_data <- radf(sim_data_wdate)

autoplot(rsim_data)

# Modify facet_wrap options through ellipsis
autoplot(rsim_data, scales = "free_y", dir = "v")

# Modify the shading options
autoplot(rsim_data, shade_opt = shade(fill = "pink", opacity = 0.5))

# Allow for nonrejected series to be plotted
autoplot(rsim_data, nonrejected = TRUE)

# Remove the shading completely (2 ways)
autoplot(rsim_data, shade_opt = NULL)
autoplot(rsim_data, shade_opt = shade(opacity = 0))

# Plot only the series with the shading options
autoplot2(rsim_data)
autoplot2(rsim_data, trunc = FALSE) # keep the minw period

# We will need ggplot2 from here on out
library(ggplot2)

# Change (overwrite) color, size or linetype
autoplot(rsim_data) +
  scale_color_manual(values = c("black", "black")) +
  scale_size_manual(values = c(0.9, 1)) +
  scale_linetype_manual(values = c("solid", "solid"))

# Change names through labeller (first way)
custom_labels <- c("psy1" = "new_name_for_psy1", "psy2" = "new_name_for_psy2")
autoplot(rsim_data, labeller = labeller(.default = label_value, id = as_labeller(custom_labels)))

# Change names through labeller (second way)
custom_labels2 <- series_names(rsim_data)
names(custom_labels2) <- custom_labels2
custom_labels2[c(3,5)] <- c("Evans", "Blanchard")
autoplot(rsim_data, labeller = labeller(id = custom_labels2))

# Or change names before plotting
series_names(rsim_data) <- LETTERS[1:5]
```

```

autoplot(rsim_data)

# Change Theme options
autoplot(rsim_data) +
  theme(legend.position = "right")

```

---

datestamp

*Date-stamping periods of mildly explosive behavior*


---

### Description

Computes the origination, termination and duration of episodes during which the time series display explosive dynamics.

### Usage

```
datestamp(object, cv = NULL, min_duration = 0L, ...)
```

```

## S3 method for class 'radf_obj'
datestamp(
  object,
  cv = NULL,
  min_duration = 0L,
  sig_lvl = 95,
  option = c("gsadf", "sadf"),
  nonrejected = FALSE,
  ...
)

```

### Arguments

|              |                                                                                                                  |
|--------------|------------------------------------------------------------------------------------------------------------------|
| object       | An object of class obj.                                                                                          |
| cv           | An object of class cv.                                                                                           |
| min_duration | The minimum duration of an explosive period for it to be reported (default = 0).                                 |
| ...          | further arguments passed to methods.                                                                             |
| sig_lvl      | logical. Significance level, one of 90, 95 or 99.                                                                |
| option       | Whether to apply the "gsadf" or "sadf" methodology (default = "gsadf").                                          |
| nonrejected  | logical. Whether to apply datestamping technique to the series that were not able to reject the Null hypothesis. |

### Details

Datestamp also stores a vector whose elements take the value of 1 when there is a period of explosive behaviour and 0 otherwise. This output can serve as a dummy variable for the occurrence of exuberance.

**Value**

Return a table with the following columns:

- Start:
- Peak:
- End:
- Duration:
- Signal:
- Ongoing:

Returns a list containing the estimated origination and termination dates of episodes of explosive behaviour and the corresponding duration.

**References**

Phillips, P. C. B., Shi, S., & Yu, J. (2015). Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500. *International Economic Review*, 56(4), 1043-1078.

**Examples**

```
rsim_data <- radf(sim_data)

ds_data <- datestamp(rsim_data)
ds_data

# Choose minimum window
datestamp(rsim_data, min_duration = psy_ds(nrow(sim_data)))

autoplot(ds_data)
```

---

diagnostics

*Diagnostics on hypothesis testing*


---

**Description**

Provides information on whether the null hypothesis of a unit root is rejected against the alternative of explosive behaviour for each series in a dataset.

**Usage**

```
diagnostics(object, cv = NULL, ...)

## S3 method for class 'radf_obj'
diagnostics(object, cv = NULL, option = c("gsadf", "sadf"), ...)
```

**Arguments**

|        |                                                                         |
|--------|-------------------------------------------------------------------------|
| object | An object of class obj.                                                 |
| cv     | An object of class cv.                                                  |
| ...    | Further arguments passed to methods.                                    |
| option | Whether to apply the "gsadf" or "sadf" methodology (default = "gsadf"). |

**Details**

Diagnostics also stores a vector whose elements take the value of 1 when there is a period of explosive behaviour and 0 otherwise.

**Value**

Returns a list with the series that reject (positive) and the series that do not reject (negative) the null hypothesis, and at what significance level.

**Examples**

```
rsim_data <- radf(sim_data)
diagnostics(rsim_data)

diagnostics(rsim_data, option = "sadf")
```

---

|          |                                   |
|----------|-----------------------------------|
| index-rd | <i>Retrieve/Replace the index</i> |
|----------|-----------------------------------|

---

**Description**

Retrieve or replace the index of an object.

**Usage**

```
index(x, ...)
```

```
index(x) <- value
```

**Arguments**

|       |                                                                     |
|-------|---------------------------------------------------------------------|
| x     | An object.                                                          |
| ...   | Further arguments passed to methods.                                |
| value | An ordered vector of the same length as the 'index' attribute of x. |

**Details**

If the user does not specify an index for the estimation a pseudo-index is generated which is a sequential numeric series. After the estimation, the user can use index to retrieve or ``index<-`` to replace the index. The index can be either numeric or Date.

---

|                    |                                   |
|--------------------|-----------------------------------|
| install_exuberdata | <i>Install exuberdata Package</i> |
|--------------------|-----------------------------------|

---

### Description

This function wraps the `install.packages` function and offers a faster and more convenient way to install `exuberdata`.

### Usage

```
install_exuberdata()
```

### Examples

```
if("exuberdata" %in% loadedNamespaces()) {
  exuberdata::radf_crit2
}
```

---

|          |                                                    |
|----------|----------------------------------------------------|
| psy_minw | <i>Helper functions in accordance to PSY(2015)</i> |
|----------|----------------------------------------------------|

---

### Description

`psy_minw` and `psy_ds` use the rules-of-thumb proposed by Phillips et al. (2015) to compute the minimum window size and the minimum duration of an episode of exuberance, respectively.

### Usage

```
psy_minw(n)
```

```
psy_ds(n, rule = 1, delta = 1)
```

### Arguments

|                    |                                                                                                                                                                                          |
|--------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>     | A positive integer. The sample size.                                                                                                                                                     |
| <code>rule</code>  | Rule to compute the minimum duration of an episode (default: <code>rule = 1</code> , where $T$ denotes the sample size). Rule 1 corresponds to $\log(T)$ , while rule 2 to $\log(T)/T$ . |
| <code>delta</code> | Frequency-dependent parameter (default; <code>delta = 1</code> ). See details.                                                                                                           |

### Details

For the minimum duration period, `psy_ds` allows the user to choose from two rules:

$$rule_1 = \delta \log(n) \quad \& \quad rule_2 = \delta \log(n)/n$$

`delta` depends on the frequency of the data and the minimal duration condition.

## References

Phillips, P. C. B., Shi, S., & Yu, J. (2015). Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500. *International Economic Review*, 56(4), 1043-1078.

## Examples

```
psy_minw(100)
psy_ds(100)
```

---

ps\_tb

*Helper function to find tb from the Phillips and Shi (2020)*

---

## Description

This function helps to find the number of observations in the window over which size is to be controlled.

## Usage

```
ps_tb(n, freq = c("monthly", "quarterly", "annual", "weekly"), size = 2)
```

## Arguments

|      |                                      |
|------|--------------------------------------|
| n    | A positive integer. The sample size. |
| freq | The type of date-interval.           |
| size | The size to be controlled.           |

## References

Phillips, P. C., & Shi, S. (2020). Real time monitoring of asset markets: Bubbles and crises. In *Handbook of Statistics* (Vol. 42, pp. 61-80). Elsevier.

Shi, S., Hurn, S., Phillips, P.C.B., 2018. Causal change detection in possibly integrated systems: Revisiting the money-income relationship.

radf

*Recursive Augmented Dickey-Fuller Test***Description**

radf returns the recursive univariate and panel Augmented Dickey-Fuller test statistics.

**Usage**

```
radf(data, minw = NULL, lag = 0L)
```

**Arguments**

|      |                                                                                                                                                 |
|------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| data | A univariate or multivariate numeric time series object, a numeric vector or matrix, or a data.frame. The object should not have any NA values. |
| minw | A positive integer. The minimum window size (default = $(0.01 + 1.8/\sqrt{T})T$ , where T denotes the sample size).                             |
| lag  | A non-negative integer. The lag length of the Augmented Dickey-Fuller regression (default = 0L).                                                |

**Details**

The radf() function is vectorized, i.e., it can handle multiple series at once, to improve efficiency. This property also enables the computation of panel statistics internally as a by-product of the univariate estimations with minimal additional cost incurred.

**Value**

A list that contains the unit root test statistics (sequence):

|             |                                                    |
|-------------|----------------------------------------------------|
| adf         | Augmented Dickey-Fuller                            |
| badf        | Backward Augmented Dickey-Fuller                   |
| sadf        | Supremum Augmented Dickey-Fuller                   |
| bsadf       | Backward Supremum Augmented Dickey-Fuller          |
| gsadf       | Generalized Supremum Augmented Dickey-Fuller       |
| bsadf_panel | Panel Backward Supremum Augmented Dickey-Fuller    |
| gsadf_panel | Panel Generalized Supremum Augmented Dickey-Fuller |

And attributes:

|              |                                            |
|--------------|--------------------------------------------|
| mat          | The matrix used in the estimation.         |
| index        | The index parsed from the dataset.         |
| lag          | The lag used in the estimation.            |
| n            | The number of rows.                        |
| minw         | The minimum window used in the estimation. |
| series_names | The series names.                          |

## References

- Phillips, P. C. B., Wu, Y., & Yu, J. (2011). Explosive Behavior in The 1990s Nasdaq: When Did Exuberance Escalate Asset Values? *International Economic Review*, 52(1), 201-226.
- Phillips, P. C. B., Shi, S., & Yu, J. (2015). Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500. *International Economic Review*, 56(4), 1043-1078.
- Pavlidis, E., Yusupova, A., Paya, I., Peel, D., Martínez-García, E., Mack, A., & Grossman, V. (2016). Episodes of exuberance in housing markets: in search of the smoking gun. *The Journal of Real Estate Finance and Economics*, 53(4), 419-449.

## Examples

```
# We will use simulated data that are stored as data
sim_data

rsim <- radf(sim_data)

str(rsim)

# We would also use data that contain a Date column
sim_data_wdate

rsim_wdate <- radf(sim_data_wdate)

tidy(rsim_wdate)

augment(rsim_wdate)

tidy(rsim_wdate, panel = TRUE)

head(index(rsim_wdate))

# For lag = 1 and minimum window = 20
rsim_20 <- radf(sim_data, minw = 20, lag = 1)
```

---

radf\_crit

*Stored Monte Carlo Critical Values*


---

## Description

A dataset containing Monte Carlo critical values for up to 600 observations generated using the default minimum window. The critical values have been simulated and stored as data to save computation time for the user. The stored critical values can be obtained with the `radf_mc_cv()` function, using `nrep = 2000` and the seed = 123.

## Usage

```
radf_crit
```

**Format**

A list with lower level lists that contain

**adf\_cv**: Augmented Dickey-Fuller

**badf\_cv**: Backward Augmented Dickey-Fuller

**sadf\_cv**: Supremum Augmented Dickey-Fuller

**bsadf\_cv**: Backward Supremum Augmented Dickey-Fuller

**gsadf\_cv**: Generalized Supremum Augmented Dickey Fuller

**Source**

Simulated from exuber package function [radf\\_mc\\_cv\(\)](#).

**Examples**

```
## Not run:
all.equal(radf_crit[[50]], radf_mc_cv(50, nrep = 2000, seed = 123))

## End(Not run)
```

---

radf\_mc\_cv

*Monte Carlo Critical Values*


---

**Description**

radf\_mc\_cv computes Monte Carlo critical values for the recursive unit root tests. radf\_mc\_distr computes the distribution.

**Usage**

```
radf_mc_cv(n, minw = NULL, nrep = 1000L, seed = NULL)
```

```
radf_mc_distr(n, minw = NULL, nrep = 1000L, seed = NULL)
```

**Arguments**

|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n    | A positive integer. The sample size.                                                                                                                                                                                                                                                                                                                                                                                                               |
| minw | A positive integer. The minimum window size (default = $(0.01 + 1.8/\sqrt{T})T$ , where T denotes the sample size).                                                                                                                                                                                                                                                                                                                                |
| nrep | A positive integer. The number of Monte Carlo simulations.                                                                                                                                                                                                                                                                                                                                                                                         |
| seed | An object specifying if and how the random number generator (rng) should be initialized. Either NULL or an integer will be used in a call to set.seed before simulation. If set, the value is saved as "seed" attribute of the returned value. The default, NULL, will not change rng state, and return .Random.seed as the "seed" attribute. Results are different between the parallel and non-parallel option, even if they have the same seed. |

**Value**

For `radf_mc_cv` a list that contains the critical values for ADF, BADF, BSADF and GSADF test statistics. For `radf_mc_distr` a list that contains the ADF, SADF and GSADF distributions.

**See Also**

[radf\\_wb\\_cv](#) for wild bootstrap critical values and [radf\\_sb\\_cv](#) for sieve bootstrap critical values

**Examples**

```
# Default minimum window
mc <- radf_mc_cv(n = 100)

tidy(mc)

# Change the minimum window and the number of simulations
mc2 <- radf_mc_cv(n = 100, nrep = 600, minw = 20)

tidy(mc2)

mdist <- radf_mc_distr(n = 100, nrep = 1000)

autoplot(mdist)
```

---

radf\_sb\_cv

*Panel Sieve Bootstrap Critical Values*


---

**Description**

`radf_sb_cv` computes critical values for the panel recursive unit root test using the sieve bootstrap procedure outlined in Pavlidis et al. (2016). `radf_sb_distr` computes the distribution.

**Usage**

```
radf_sb_cv(data, minw = NULL, lag = 0L, nboot = 500L, seed = NULL)

radf_sb_distr(data, minw = NULL, lag = 0L, nboot = 500L, seed = NULL)
```

**Arguments**

|                   |                                                                                                                                                 |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>data</code> | A univariate or multivariate numeric time series object, a numeric vector or matrix, or a data.frame. The object should not have any NA values. |
| <code>minw</code> | A positive integer. The minimum window size (default = $(0.01 + 1.8/\sqrt{T})T$ , where $T$ denotes the sample size).                           |
| <code>lag</code>  | A non-negative integer. The lag length of the Augmented Dickey-Fuller regression (default = 0L).                                                |

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| nboot | A positive integer. Number of bootstraps (default = 500L).                                                                                                                                                                                                                                                                                                                                                                                                                   |
| seed  | An object specifying if and how the random number generator (rng) should be initialized. Either NULL or an integer will be used in a call to <code>set.seed</code> before simulation. If set, the value is saved as "seed" attribute of the returned value. The default, NULL, will not change rng state, and return <code>.Random.seed</code> as the "seed" attribute. Results are different between the parallel and non-parallel option, even if they have the same seed. |

### Value

For `radf_sb_cv` A list A list that contains the critical values for the panel BSADF and panel GSADF test statistics. For `radf_wb_dist` a numeric vector that contains the distribution of the panel GSADF statistic.

### References

Pavlidis, E., Yusupova, A., Paya, I., Peel, D., Martínez-García, E., Mack, A., & Grossman, V. (2016). Episodes of exuberance in housing markets: In search of the smoking gun. *The Journal of Real Estate Finance and Economics*, 53(4), 419-449.

### See Also

[radf\\_mc\\_cv](#) for Monte Carlo critical values and [radf\\_wb\\_cv](#) for wild Bootstrap critical values

### Examples

```
rsim_data <- radf(sim_data, lag = 1)

# Critical vales should have the same lag length with \code{radf()}
sb <- radf_sb_cv(sim_data, lag = 1)

tidy(sb)

summary(rsim_data, cv = sb)

autoplot(rsim_data, cv = sb)

# Simulate distribution
sdist <- radf_sb_distr(sim_data, lag = 1, nboot = 1000)

autoplot(sdist)
```

---

|            |                                       |
|------------|---------------------------------------|
| radf_wb_cv | <i>Wild Bootstrap Critical Values</i> |
|------------|---------------------------------------|

---

**Description**

radf\_wb\_cv performs the Harvey et al. (2016) wild bootstrap re-sampling scheme, which is asymptotically robust to non-stationary volatility, to generate critical values for the recursive unit root tests. radf\_wb\_distr computes the distribution.

**Usage**

```
radf_wb_cv(data, minw = NULL, nboot = 500L, dist_rad = FALSE, seed = NULL)
```

```
radf_wb_distr(data, minw = NULL, nboot = 500L, dist_rad = FALSE, seed = NULL)
```

**Arguments**

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| data     | A univariate or multivariate numeric time series object, a numeric vector or matrix, or a data.frame. The object should not have any NA values.                                                                                                                                                                                                                                                                                                                              |
| minw     | A positive integer. The minimum window size (default = $(0.01 + 1.8/\sqrt{T})T$ , where T denotes the sample size).                                                                                                                                                                                                                                                                                                                                                          |
| nboot    | A positive integer. Number of bootstraps (default = 500L).                                                                                                                                                                                                                                                                                                                                                                                                                   |
| dist_rad | Logical. If TRUE then the Rademacher distribution will be used.                                                                                                                                                                                                                                                                                                                                                                                                              |
| seed     | An object specifying if and how the random number generator (rng) should be initialized. Either NULL or an integer will be used in a call to <code>set.seed</code> before simulation. If set, the value is saved as "seed" attribute of the returned value. The default, NULL, will not change rng state, and return <code>.Random.seed</code> as the "seed" attribute. Results are different between the parallel and non-parallel option, even if they have the same seed. |

**Details**

This approach involves applying a wild bootstrap re-sampling scheme to construct the bootstrap analogue of the Phillips et al. (2015) test which is asymptotically robust to non-stationary volatility.

**Value**

For `radf_wb_cv` a list that contains the critical values for the ADF, BADF, BSADF and GSADF tests. For `radf_wb_distr` a list that contains the ADF, SADF and GSADF distributions.

**References**

Harvey, D. I., Leybourne, S. J., Sollis, R., & Taylor, A. M. R. (2016). Tests for explosive financial bubbles in the presence of non-stationary volatility. *Journal of Empirical Finance*, 38(Part B), 548-574.

Phillips, P. C. B., Shi, S., & Yu, J. (2015). Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500. *International Economic Review*, 56(4), 1043-1078.

**See Also**

[radf\\_mc\\_cv](#) for Monte Carlo critical values and [radf\\_sb\\_cv](#) for sieve bootstrap critical values.

**Examples**

```
# Default minimum window
wb <- radf_wb_cv(sim_data)

tidy(wb)

# Change the minimum window and the number of bootstraps
wb2 <- radf_wb_cv(sim_data, nboot = 600, minw = 20)

tidy(wb2)

# Simulate distribution
wdist <- radf_wb_distr(sim_data)

autoplot(wdist)
```

---

radf\_wb\_cv2

---

*Wild Bootstrap Critical Values*


---

**Description**

`radf_wb_cv` performs the Phillips & Shi (2020) wild bootstrap re-sampling scheme, which is asymptotically robust to non-stationary volatility, to generate critical values for the recursive unit root tests. `radf_wb_distr2` computes the distribution.

**Usage**

```
radf_wb_cv2(
  data,
  minw = NULL,
  nboot = 500L,
  adflag = 0,
  type = c("fixed", "aic", "bic"),
  tb = NULL,
  seed = NULL
)

radf_wb_distr2(
  data,
  minw = NULL,
  nboot = 500L,
  adflag = 0,
  type = c("fixed", "aic", "bic"),
```

```

    tb = NULL,
    seed = NULL
  )

```

### Arguments

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|--------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| data   | A univariate or multivariate numeric time series object, a numeric vector or matrix, or a data.frame. The object should not have any NA values.                                                                                                                                                                                                                                                                                                                              |
| minw   | A positive integer. The minimum window size (default = $(0.01 + 1.8/\sqrt{T})T$ , where T denotes the sample size).                                                                                                                                                                                                                                                                                                                                                          |
| nboot  | A positive integer. Number of bootstraps (default = 500L).                                                                                                                                                                                                                                                                                                                                                                                                                   |
| adflag | A positive integer. Number of lags when type is "fixed" or number of max lags when type is either "aic" or "bic".                                                                                                                                                                                                                                                                                                                                                            |
| type   | Character. "fixed" for fixed lag, "aic" or "bic" for automatic lag selection according to the criterion.                                                                                                                                                                                                                                                                                                                                                                     |
| tb     | A positive integer. The simulated sample size.                                                                                                                                                                                                                                                                                                                                                                                                                               |
| seed   | An object specifying if and how the random number generator (rng) should be initialized. Either NULL or an integer will be used in a call to <code>set.seed</code> before simulation. If set, the value is saved as "seed" attribute of the returned value. The default, NULL, will not change rng state, and return <code>.Random.seed</code> as the "seed" attribute. Results are different between the parallel and non-parallel option, even if they have the same seed. |

### Value

For `radf_wb_cv2` a list that contains the critical values for the ADF, BADF, BSADF and GSADF tests. For `radf_wb_distr` a list that contains the ADF, SADF and GSADF distributions.

### References

- Phillips, P. C., & Shi, S. (2020). Real time monitoring of asset markets: Bubbles and crises. In *Handbook of Statistics* (Vol. 42, pp. 61-80). Elsevier.
- Phillips, P. C. B., Shi, S., & Yu, J. (2015). Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500. *International Economic Review*, 56(4), 1043-1078.

### See Also

[radf\\_mc\\_cv](#) for Monte Carlo critical values and [radf\\_sb\\_cv](#) for sieve bootstrap critical values.

### Examples

```

# Default minimum window
wb <- radf_wb_cv2(sim_data)

tidy(wb)

# Change the minimum window and the number of bootstraps
wb2 <- radf_wb_cv2(sim_data, nboot = 600, minw = 20)

```

```

tidy(wb2)

# Simulate distribution
wdist <- radf_wb_distr(sim_data)

autoplot(wdist)

```

---

scale\_exuber\_manual      *Exuber scale and theme functions*

---

## Description

scale\_exuber\_manual allows specifying the color, size and linetype in autoplot.radf\_obj mappings. theme\_exuber is a complete theme which control all non-data display.

## Usage

```

scale_exuber_manual(
  color_values = c("red", "blue"),
  linetype_values = c(2, 1),
  size_values = c(0.8, 0.7)
)

theme_exuber(
  base_size = 11,
  base_family = "",
  base_line_size = base_size/22,
  base_rect_size = base_size/22
)

```

## Arguments

|                 |                                                 |
|-----------------|-------------------------------------------------|
| color_values    | a set of color values to map data values to.    |
| linetype_values | a set of linetype values to map data values to. |
| size_values     | a set of size values to map data values to.     |
| base_size       | base font size, given in pts.                   |
| base_family     | base font family                                |
| base_line_size  | base size for line elements                     |
| base_rect_size  | base size for rect elements                     |

---

|              |                                      |
|--------------|--------------------------------------|
| series_names | <i>Retrieve/Replace series names</i> |
|--------------|--------------------------------------|

---

**Description**

Retrieve or replace the series names of an object.

**Usage**

```
series_names(x, ...)

series_names(x) <- value

## S3 replacement method for class 'radf_obj'
series_names(x) <- value

## S3 replacement method for class 'wb_cv'
series_names(x) <- value

## S3 replacement method for class 'sb_cv'
series_names(x) <- value
```

**Arguments**

|       |                                                                    |
|-------|--------------------------------------------------------------------|
| x     | An object.                                                         |
| ...   | Further arguments passed to methods.                               |
| value | n ordered vector of the same length as the "index" attribute of x. |

**Examples**

```
# Simulate bubble processes
dta <- data.frame(psy1 = sim_psy1(n = 100), psy2 = sim_psy2(n = 100))

rfd <- radf(dta)

series_names(rfd) <- c("OneBubble", "TwoBubbles")
```

---

|          |                                                        |
|----------|--------------------------------------------------------|
| sim_blan | <i>Simulation of a Blanchard (1979) bubble process</i> |
|----------|--------------------------------------------------------|

---

**Description**

Simulation of a Blanchard (1979) rational bubble process.

**Usage**

```
sim_blan(n, pi = 0.7, sigma = 0.03, r = 0.05, b0 = 0.1, seed = NULL)
```

**Arguments**

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n     | A positive integer specifying the length of the simulated output series.                                                                                                                                                                                                                                                                                                                                                                                                     |
| pi    | A positive value in (0, 1) which governs the probability of the bubble continuing to grow.                                                                                                                                                                                                                                                                                                                                                                                   |
| sigma | A positive scalar indicating the standard deviation of the innovations.                                                                                                                                                                                                                                                                                                                                                                                                      |
| r     | A positive scalar that determines the growth rate of the bubble process.                                                                                                                                                                                                                                                                                                                                                                                                     |
| b0    | The initial value of the bubble.                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| seed  | An object specifying if and how the random number generator (rng) should be initialized. Either NULL or an integer will be used in a call to <code>set.seed</code> before simulation. If set, the value is saved as "seed" attribute of the returned value. The default, NULL, will not change rng state, and return <code>.Random.seed</code> as the "seed" attribute. Results are different between the parallel and non-parallel option, even if they have the same seed. |

**Details**

Blanchard's bubble process has two regimes, which occur with probability  $\pi$  and  $1 - \pi$ . In the first regime, the bubble grows exponentially, whereas in the second regime, the bubble collapses to a white noise.

With probability  $\pi$ :

$$B_{t+1} = \frac{1+r}{\pi} B_t + \epsilon_{t+1}$$

With probability  $1 - \pi$ :

$$B_{t+1} = \epsilon_{t+1}$$

where  $r$  is a positive constant and  $\epsilon \sim iid(0, \sigma^2)$ .

**Value**

A numeric vector of length  $n$ .

**References**

Blanchard, O. J. (1979). Speculative bubbles, crashes and rational expectations. *Economics letters*, 3(4), 387-389.

**See Also**

[sim\\_psy1](#), [sim\\_psy2](#), [sim\\_evans](#)

**Examples**

```
sim_blan(n = 100, seed = 123) %>%
  autoplot()
```

sim\_div

*Simulation of dividends***Description**

Simulate (log) dividends from a random walk with drift.

**Usage**

```
sim_div(
  n,
  mu,
  sigma,
  r = 0.05,
  log = FALSE,
  output = c("pf", "d"),
  seed = NULL
)
```

**Arguments**

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|--------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n      | A positive integer specifying the length of the simulated output series.                                                                                                                                                                                                                                                                                                                                                                                                     |
| mu     | A scalar indicating the drift.                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| sigma  | A positive scalar indicating the standard deviation of the innovations.                                                                                                                                                                                                                                                                                                                                                                                                      |
| r      | A positive value indicating the discount factor.                                                                                                                                                                                                                                                                                                                                                                                                                             |
| log    | Logical. If true dividends follow a lognormal distribution.                                                                                                                                                                                                                                                                                                                                                                                                                  |
| output | A character string giving the fundamental price("pf") or dividend series("d"). Default is 'pf'.                                                                                                                                                                                                                                                                                                                                                                              |
| seed   | An object specifying if and how the random number generator (rng) should be initialized. Either NULL or an integer will be used in a call to <code>set.seed</code> before simulation. If set, the value is saved as "seed" attribute of the returned value. The default, NULL, will not change rng state, and return <code>.Random.seed</code> as the "seed" attribute. Results are different between the parallel and non-parallel option, even if they have the same seed. |

**Details**

If log is set to FALSE (default value) dividends follow:

$$d_t = \mu + d_{t-1} + \epsilon_t$$

where  $\epsilon \sim \mathcal{N}(0, \sigma^2)$ . The default parameters are  $\mu = 0.0373$ ,  $\sigma^2 = 0.1574$  and  $d[0] = 1.3$  (the initial value of the dividend sequence). The above equation can be solved to yield the fundamental price:

$$F_t = \mu(1+r)r^{-2} + r^{-1}d_t$$

If log is set to TRUE then dividends follow a lognormal distribution or log(dividends) follow:

$$\ln(d_t) = \mu + \ln(d_{t-1}) + \epsilon_t$$

where  $\epsilon \sim \mathcal{N}(0, \sigma^2)$ . Default parameters are  $\mu = 0.013$ ,  $\sigma^2 = 0.16$ . The fundamental price in this case is:

$$F_t = \frac{1+g}{r-g}d_t$$

where  $1+g = \exp(\mu + \sigma^2/2)$ . All default parameter values are those suggested by West (1988).

### Value

A numeric vector of length n.

### References

West, K. D. (1988). Dividend innovations and stock price volatility. *Econometrica: Journal of the Econometric Society*, p. 37-61.

### Examples

```
# Price is the sum of the bubble and fundamental components
# 20 is the scaling factor
pf <- sim_div(100, r = 0.05, output = "pf", seed = 123)
pb <- sim_evans(100, r = 0.05, seed = 123)
p <- pf + 20 * pb

autoplot(p)
```

---

sim\_evans

*Simulation of an Evans (1991) bubble process*

---

### Description

Simulation of an Evans (1991) rational periodically collapsing bubble process.

**Usage**

```
sim_evans(
  n,
  alpha = 1,
  delta = 0.5,
  tau = 0.05,
  pi = 0.7,
  r = 0.05,
  b1 = delta,
  seed = NULL
)
```

**Arguments**

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n     | A positive integer specifying the length of the simulated output series.                                                                                                                                                                                                                                                                                                                                                                                                     |
| alpha | A positive scalar, with restrictions (see details).                                                                                                                                                                                                                                                                                                                                                                                                                          |
| delta | A positive scalar, with restrictions (see details).                                                                                                                                                                                                                                                                                                                                                                                                                          |
| tau   | The standard deviation of the innovations.                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| pi    | A positive value in (0, 1) which governs the probability of the bubble continuing to grow.                                                                                                                                                                                                                                                                                                                                                                                   |
| r     | A positive scalar that determines the growth rate of the bubble process.                                                                                                                                                                                                                                                                                                                                                                                                     |
| b1    | A positive scalar, the initial value of the series. Defaults to delta.                                                                                                                                                                                                                                                                                                                                                                                                       |
| seed  | An object specifying if and how the random number generator (rng) should be initialized. Either NULL or an integer will be used in a call to <code>set.seed</code> before simulation. If set, the value is saved as "seed" attribute of the returned value. The default, NULL, will not change rng state, and return <code>.Random.seed</code> as the "seed" attribute. Results are different between the parallel and non-parallel option, even if they have the same seed. |

**Details**

delta and alpha are positive parameters which satisfy  $0 < \delta < (1 + r)\alpha$ . delta represents the size of the bubble after collapse. The default value of r is 0.05. The function checks whether alpha and delta satisfy this condition and will return an error if not.

The Evans bubble has two regimes. If  $B_t \leq \alpha$  the bubble grows at an average rate of  $1 + r$ :

$$B_{t+1} = (1 + r)B_t u_{t+1},$$

When  $B_t > \alpha$  the bubble expands at the increased rate of  $(1 + r)\pi^{-1}$ :

$$B_{t+1} = [\delta + (1 + r)\pi^{-1}\theta_{t+1}(B_t - (1 + r)^{-1}\delta B_t)]u_{t+1},$$

where  $\theta$  theta is a binary variable that takes the value 0 with probability  $1 - \pi$  and 1 with probability  $\pi$ . In the second phase, there is a  $(1 - \pi)$  probability of the bubble process collapsing to delta. By modifying the values of delta, alpha and pi the user can change the frequency at which bubbles appear, the mean duration of a bubble before collapse and the scale of the bubble.

**Value**

A numeric vector of length  $n$ .

**References**

Evans, G. W. (1991). Pitfalls in testing for explosive bubbles in asset prices. The American Economic Review, 81(4), 922-930.

**See Also**

[sim\\_psy1](#), [sim\\_psy2](#), [sim\\_blan](#)

**Examples**

```
sim_evans(100, seed = 123) %>%
  autoplot()
```

---

|         |                                                                                     |
|---------|-------------------------------------------------------------------------------------|
| sim_ps1 | <i>Simulation of a single-bubble process with multiple forms of collapse regime</i> |
|---------|-------------------------------------------------------------------------------------|

---

**Description**

The new generating process considered here differs from the `sim_psy1` model in three respects - Phillips and Shi (2018):

*First, it includes an asymptotically negligible drift in the martingale path during normal periods. Second, the collapse process is modeled directly as a transient mildly integrated process that covers an explicit period of market collapse. Third, a market recovery date is introduced to capture the return to normal market behavior.*

- sudden: with  $\beta = 0.1$  and  $tr = tf + 0.01 \cdot n$
- disturbing: with  $\beta = 0.5$  and  $tr = tf + 0.1 \cdot n$
- smooth: with  $\beta = 0.9$  and  $tr = tf + 0.2 \cdot n$

In order to provide the duration of the collapse period  $tr$  as  $tr = tf + 0.2n$ , you have to provide  $tf$  as well.

**Usage**

```
sim_ps1(
  n,
  te = 0.4 * n,
  tf = te + 0.2 * n,
  tr = tf + 0.1 * n,
  c = 1,
  c1 = 1,
  c2 = 1,
```

```

    eta = 0.6,
    alpha = 0.6,
    beta = 0.5,
    sigma = 6.79,
    seed = NULL
)

```

### Arguments

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n     | A positive integer specifying the length of the simulated output series.                                                                                                                                                                                                                                                                                                                                                                                                     |
| te    | A scalar in (0, tf) specifying the observation in which the bubble originates.                                                                                                                                                                                                                                                                                                                                                                                               |
| tf    | A scalar in (te, n) specifying the observation in which the bubble collapses.                                                                                                                                                                                                                                                                                                                                                                                                |
| tr    | A scalar in (tf, n) specifying the observation in which market recovers                                                                                                                                                                                                                                                                                                                                                                                                      |
| c     | A positive scalar determining the drift in the normal market periods.                                                                                                                                                                                                                                                                                                                                                                                                        |
| c1    | A positive scalar determining the autoregressive coefficient in the explosive regime.                                                                                                                                                                                                                                                                                                                                                                                        |
| c2    | A positive scalar determining the autoregressive coefficient in the collapse regime.                                                                                                                                                                                                                                                                                                                                                                                         |
| eta   | A positive scalar (>0.5) determining the drift in the normal market periods.                                                                                                                                                                                                                                                                                                                                                                                                 |
| alpha | A positive scalar in (0, 1) determining the autoregressive coefficient in the bubble period.                                                                                                                                                                                                                                                                                                                                                                                 |
| beta  | A positive scalar in (0, 1) determining the autoregressive coefficient in the collapse period.                                                                                                                                                                                                                                                                                                                                                                               |
| sigma | A positive scalar indicating the standard deviation of the innovations.                                                                                                                                                                                                                                                                                                                                                                                                      |
| seed  | An object specifying if and how the random number generator (rng) should be initialized. Either NULL or an integer will be used in a call to <code>set.seed</code> before simulation. If set, the value is saved as "seed" attribute of the returned value. The default, NULL, will not change rng state, and return <code>.Random.seed</code> as the "seed" attribute. Results are different between the parallel and non-parallel option, even if they have the same seed. |

### Value

A numeric vector of length n.

### References

Phillips, Peter CB, and Shu-Ping Shi. "Financial bubble implosion and reverse regression." *Econometric Theory* 34.4 (2018): 705-753.

### See Also

[sim\\_psy1](#)

## Examples

```
# Disturbing collapse (default)
disturbing <- sim_psy1(100)
autoplot(disturbing)

# Sudden collapse
sudden <- sim_psy1(100, te = 40, tf= 60, tr = 61, beta = 0.1)
autoplot(sudden)
```

---

sim\_psy1

*Simulation of a single-bubble process*


---

## Description

The following function generates a time series which switches from a martingale to a mildly explosive process and then back to a martingale.

## Usage

```
sim_psy1(
  n,
  te = 0.4 * n,
  tf = 0.15 * n + te,
  c = 1,
  alpha = 0.6,
  sigma = 6.79,
  seed = NULL
)
```

## Arguments

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n     | A positive integer specifying the length of the simulated output series.                                                                                                                                                                                                                                                                                                                                                                                                     |
| te    | A scalar in (0, tf) specifying the observation in which the bubble originates.                                                                                                                                                                                                                                                                                                                                                                                               |
| tf    | A scalar in (te, n) specifying the observation in which the bubble collapses.                                                                                                                                                                                                                                                                                                                                                                                                |
| c     | A positive scalar determining the autoregressive coefficient in the explosive regime.                                                                                                                                                                                                                                                                                                                                                                                        |
| alpha | A positive scalar in (0, 1) determining the value of the expansion rate in the autoregressive coefficient.                                                                                                                                                                                                                                                                                                                                                                   |
| sigma | A positive scalar indicating the standard deviation of the innovations.                                                                                                                                                                                                                                                                                                                                                                                                      |
| seed  | An object specifying if and how the random number generator (rng) should be initialized. Either NULL or an integer will be used in a call to <code>set.seed</code> before simulation. If set, the value is saved as "seed" attribute of the returned value. The default, NULL, will not change rng state, and return <code>.Random.seed</code> as the "seed" attribute. Results are different between the parallel and non-parallel option, even if they have the same seed. |

## Details

The data generating process is described by the following equation:

$$X_t = X_{t-1}1\{t < \tau_e\} + \delta_T X_{t-1}1\{\tau_e \leq t \leq \tau_f\} + \left( \sum_{k=\tau_f+1}^t \epsilon_k + X_{\tau_f} \right) 1\{t > \tau_f\} + \epsilon_t 1\{t \leq \tau_f\}$$

where the autoregressive coefficient  $\delta_T$  is given by:

$$\delta_T = 1 + cT^{-\alpha}$$

with  $c > 0$ ,  $\alpha \in (0, 1)$ ,  $\epsilon \sim iid(0, \sigma^2)$  and  $X_{\tau_f} = X_{\tau_e} + X'$  with  $X' = O_p(1)$ ,  $\tau_e = [Tr_e]$  dates the origination of the bubble, and  $\tau_f = [Tr_f]$  dates the collapse of the bubble. During the pre- and post- bubble periods,  $[1, \tau_e)$ ,  $X_t$  is a pure random walk process. During the bubble expansion period  $\tau_e, \tau_f]$  becomes a mildly explosive process with expansion rate given by the autoregressive coefficient  $\delta_T$ ; and, finally during the post-bubble period,  $(\tau_f, \tau]$   $X_t$  reverts to a martingale.

For further details see Phillips et al. (2015) p. 1054.

## Value

A numeric vector of length n.

## References

Phillips, P. C. B., Shi, S., & Yu, J. (2015). Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500. *International Economic Review*, 5 6(4), 1043-1078.

## See Also

[sim\\_psy2](#), [sim\\_blan](#), [sim\\_evans](#)

## Examples

```
# 100 periods with bubble origination date 40 and termination date 55
sim_psy1(n = 100, seed = 123) %>%
  autoplot()

# 200 periods with bubble origination date 80 and termination date 110
sim_psy1(n = 200, seed = 123) %>%
  autoplot()

# 200 periods with bubble origination date 100 and termination date 150
sim_psy1(n = 200, te = 100, tf = 150, seed = 123) %>%
  autoplot()
```

sim\_psy2

*Simulation of a two-bubble process***Description**

The following data generating process is similar to [sim\\_psy1](#), with the difference that there are two episodes of mildly explosive dynamics.

**Usage**

```
sim_psy2(
  n,
  te1 = 0.2 * n,
  tf1 = 0.2 * n + te1,
  te2 = 0.6 * n,
  tf2 = 0.1 * n + te2,
  c = 1,
  alpha = 0.6,
  sigma = 6.79,
  seed = NULL
)
```

**Arguments**

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n     | A positive integer specifying the length of the simulated output series.                                                                                                                                                                                                                                                                                                                                                                                                     |
| te1   | A scalar in (0, n) specifying the observation in which the first bubble originates.                                                                                                                                                                                                                                                                                                                                                                                          |
| tf1   | A scalar in (te1, n) specifying the observation in which the first bubble collapses.                                                                                                                                                                                                                                                                                                                                                                                         |
| te2   | A scalar in (tf1, n) specifying the observation in which the second bubble originates.                                                                                                                                                                                                                                                                                                                                                                                       |
| tf2   | A scalar in (te2, n) specifying the observation in which the second bubble collapses.                                                                                                                                                                                                                                                                                                                                                                                        |
| c     | A positive scalar determining the autoregressive coefficient in the explosive regime.                                                                                                                                                                                                                                                                                                                                                                                        |
| alpha | A positive scalar in (0, 1) determining the value of the expansion rate in the autoregressive coefficient.                                                                                                                                                                                                                                                                                                                                                                   |
| sigma | A positive scalar indicating the standard deviation of the innovations.                                                                                                                                                                                                                                                                                                                                                                                                      |
| seed  | An object specifying if and how the random number generator (rng) should be initialized. Either NULL or an integer will be used in a call to <code>set.seed</code> before simulation. If set, the value is saved as "seed" attribute of the returned value. The default, NULL, will not change rng state, and return <code>.Random.seed</code> as the "seed" attribute. Results are different between the parallel and non-parallel option, even if they have the same seed. |

## Details

The two-bubble data generating process is given by (see also sim\_psy1):

$$X_t = X_{t-1}1\{t \in N_0\} + \delta_T X_{t-1}1\{t \in B_1 \cup B_2\} + \left( \sum_{k=\tau_{1f}+1}^t \epsilon_k + X_{\tau_{1f}} \right) 1\{t \in N_1\} \\ + \left( \sum_{l=\tau_{2f}+1}^t \epsilon_l + X_{\tau_{2f}} \right) 1\{t \in N_2\} + \epsilon_t 1\{t \in N_0 \cup B_1 \cup B_2\}$$

where the autoregressive coefficient  $\delta_T$  is:

$$\delta_T = 1 + cT^{-\alpha}$$

with  $c > 0$ ,  $\alpha \in (0, 1)$ ,  $\epsilon \sim iid(0, \sigma^2)$ ,  $N_0 = [1, \tau_{1e})$ ,  $B_1 = [\tau_{1e}, \tau_{1f}]$ ,  $N_1 = (\tau_{1f}, \tau_{2e})$ ,  $B_2 = [\tau_{2e}, \tau_{2f}]$ ,  $N_2 = (\tau_{2f}, \tau]$ , where  $\tau$  is the last observation of the sample. The observations  $\tau_{1e} = [Tr_{1e}]$  and  $\tau_{1f} = [Tr_{1f}]$  are the origination and termination dates of the first bubble;  $\tau_{2e} = [Tr_{2e}]$  and  $\tau_{2f} = [Tr_{2f}]$  are the origination and termination dates of the second bubble. After the collapse of the first bubble,  $X_t$  resumes a martingale path until time  $\tau_{2e} - 1$ , and a second episode of exuberance begins at  $\tau_{2e}$ . Exuberance lasts until  $\tau_{2f}$  at which point the process collapses to a value of  $X_{\tau_{2f}}$ . The process then continues on a martingale path until the end of the sample period  $\tau$ . The duration of the first bubble is assumed to be longer than that of the second bubble, i.e.  $\tau_{1f} - \tau_{1e} > \tau_{2f} - \tau_{2e}$ .

For further details you can refer to Phillips et al., (2015) p. 1055.

## Value

A numeric vector of length n.

## References

Phillips, P. C. B., Shi, S., & Yu, J. (2015). Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500. *International Economic Review*, 56(4), 1043-1078.

## See Also

[sim\\_psy1](#), [sim\\_blan](#), [sim\\_evans](#)

## Examples

```
# 100 periods with bubble origination dates 20/60 and termination dates 40/70
sim_psy2(n = 100, seed = 123) %>%
  autoplot()

# 200 periods with bubble origination dates 40/120 and termination dates 80/140
sim_psy2(n = 200, seed = 123) %>%
  autoplot()
```

---

|                  |                                |
|------------------|--------------------------------|
| summary.radf_obj | <i>Summarizing radf models</i> |
|------------------|--------------------------------|

---

## Description

summary method for radf models that consist of radf\_obj and radf\_cv.

## Usage

```
## S3 method for class 'radf_obj'
summary(object, cv = NULL, ...)
```

## Arguments

|        |                                                                                                                                |
|--------|--------------------------------------------------------------------------------------------------------------------------------|
| object | An object of class radf_obj. The output of <code>radf()</code> .                                                               |
| cv     | An object of class radf_cv. The output of <code>radf_mc_cv()</code> , <code>radf_wb_cv()</code> or <code>radf_sb_cv()</code> . |
| ...    | Further arguments passed to methods. Not used.                                                                                 |

## Value

Returns a list of summary statistics, which include the estimated ADF, SADF, and GSADF test statistics and the corresponding critical values

## Examples

```
# Simulate bubble processes, compute the test statistics and critical values
rsim_data <- radf(sim_data)

# Summary, diagnostics and datestamp (default)
summary(rsim_data)

#Summary, diagnostics and datestamp (wild bootstrap critical values)

wb <- radf_wb_cv(sim_data)

summary(rsim_data, cv = wb)
```

---

|              |                              |
|--------------|------------------------------|
| tidy.ds_radf | <i>Tidy a ds_radf object</i> |
|--------------|------------------------------|

---

**Description**

Summarizes information about ds\_radf object.

**Usage**

```
## S3 method for class 'ds_radf'  
tidy(x, ...)
```

**Arguments**

|     |                                                |
|-----|------------------------------------------------|
| x   | An object of class ds_radf.                    |
| ... | Further arguments passed to methods. Not used. |

---

|              |                              |
|--------------|------------------------------|
| tidy.radf_cv | <i>Tidy a radf_cv object</i> |
|--------------|------------------------------|

---

**Description**

Summarizes information about radf\_cv object.

**Usage**

```
## S3 method for class 'radf_cv'  
tidy(x, format = c("wide", "long"), ...)  
  
## S3 method for class 'radf_cv'  
augment(x, format = c("wide", "long"), trunc = TRUE, ...)
```

**Arguments**

|        |                                                                                    |
|--------|------------------------------------------------------------------------------------|
| x      | An object of class radf_cv.                                                        |
| format | Long or wide format (default = "wide").                                            |
| ...    | Further arguments passed to methods. Not used.                                     |
| trunc  | Whether to remove the period of the minimum window from the plot (default = TRUE). |

**Value**

A `tibble::tibble()`

- id: The series names.
- sig: The significance level.
- name: The name of the series (when format is "long").
- crit: The critical value (when format is "long").

**Examples**

```
mc <- radf_mc_cv(100)

# Get the critical values
tidy(mc)

# Get the critical value sequences
augment(mc)
```

---

|                 |                                 |
|-----------------|---------------------------------|
| tidy.radf_distr | <i>Tidy a radf_distr object</i> |
|-----------------|---------------------------------|

---

**Description**

Summarizes information about `radf_distr` object.

**Usage**

```
## S3 method for class 'radf_distr'
tidy(x, ...)
```

**Arguments**

|                  |                                                |
|------------------|------------------------------------------------|
| <code>x</code>   | An object of class <code>radf_distr</code> .   |
| <code>...</code> | Further arguments passed to methods. Not used. |

**Value**

A `tibble::tibble()`

**Examples**

```
## Not run:
mc <- mc_cv(n = 100)

tidy(mc)

## End(Not run)
```

---

|               |                               |
|---------------|-------------------------------|
| tidy.radf_obj | <i>Tidy a radf_obj object</i> |
|---------------|-------------------------------|

---

## Description

Summarizes information about radf\_obj object.

## Usage

```
## S3 method for class 'radf_obj'
tidy(x, format = c("wide", "long"), panel = FALSE, ...)

## S3 method for class 'radf_obj'
augment(x, format = c("wide", "long"), panel = FALSE, trunc = TRUE, ...)
```

## Arguments

|        |                                                                                    |
|--------|------------------------------------------------------------------------------------|
| x      | An object of class radf_obj.                                                       |
| format | Long or wide format (default = "wide").                                            |
| panel  | If TRUE then returns the panel statistics                                          |
| ...    | Further arguments passed to methods. Not used.                                     |
| trunc  | Whether to remove the period of the minimum window from the plot (default = TRUE). |

## Value

A `tibble::tibble()`

## Examples

```
dta <- data.frame(psy1 = sim_psy1(n = 100), psy2 = sim_psy2(n = 100))

rfd <- radf(dta)

# Get the test statistic
tidy(rfd)

# Get the test statistic sequences
augment(rfd)

# Get the panel test statistic
tidy(rfd, panel = TRUE)
```

---

|           |                                |
|-----------|--------------------------------|
| tidy_join | <i>Tidy into a joint model</i> |
|-----------|--------------------------------|

---

**Description**

Tidy or augment and then join objects.

**Usage**

```
tidy_join(x, y, ...)
```

```
augment_join(x, y, ...)
```

**Arguments**

|     |                                      |
|-----|--------------------------------------|
| x   | An object of class obj.              |
| y   | An object of class cv.               |
| ... | Further arguments passed to methods. |

---

|                    |                                |
|--------------------|--------------------------------|
| tidy_join.radf_obj | <i>Tidy into a joint model</i> |
|--------------------|--------------------------------|

---

**Description**

Tidy or augment and then join objects of class radf\_obj and radf\_cv. The object of reference is the radf\_cv. For example, if panel critical values are provided the function will return the panel test statistic.

**Usage**

```
## S3 method for class 'radf_obj'
tidy_join(x, y = NULL, ...)
```

```
## S3 method for class 'radf_obj'
augment_join(x, y = NULL, trunc = TRUE, ...)
```

**Arguments**

|       |                                                                                    |
|-------|------------------------------------------------------------------------------------|
| x     | An object of class radf_obj.                                                       |
| y     | An object of class radf_cv. The output will depend on the type of critical value.  |
| ...   | Further arguments passed to methods. Not used.                                     |
| trunc | Whether to remove the period of the minimum window from the plot (default = TRUE). |

**Details**

tidy\_join also calls augment\_join when cv is of class sb\_cv.

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