

Package ‘hdrcde’

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Type Package

Title Highest Density Regions and Conditional Density Estimation

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Imports locfit, ash, ks, KernSmooth, ggplot2, RColorBrewer

LazyData yes

LazyLoad yes

Description Computation of highest density regions in one and two dimensions, kernel estimation of univariate density functions conditional on one covariate, and multimodal regression.

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URL <https://pkg.robjhyndman.com/hdrcde/>,
<https://github.com/robjhyndman/hdrcde>

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alpha	<i>Alpha</i>
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Description

A simple function to change the opacity of a color

Usage

alpha(color, alpha)

Arguments

- | | |
|-------|---|
| color | the name or idea of a R color |
| alpha | a value in [0,1] defining the opacity wanted. |

BoxCox	<i>Box Cox Transformation</i>
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Description

BoxCox() returns a transformation of the input variable using a Box-Cox transformation. InvBoxCox() reverses the transformation.

Usage

```
BoxCox(x, lambda)
```

Arguments

x	a numeric vector or time series
lambda	transformation parameter

Details

The Box-Cox transformation is given by

$$f_{\lambda}(x) = \frac{x^{\lambda} - 1}{\lambda}$$

if $\lambda \neq 0$. For $\lambda = 0$,

$$f_0(x) = \log(x).$$

Value

a numeric vector of the same length as x.

Author(s)

Rob J Hyndman

References

Box, G. E. P. and Cox, D. R. (1964) An analysis of transformations. *JRSS B* **26** 211–246.

Description

Calculates kernel conditional density estimate using local polynomial estimation.

Usage

```
cde(
  x,
  y,
  deg = 0,
  link = "identity",
  a,
  b,
  mean = NULL,
  x.margin,
  y.margin,
  x.name,
  y.name,
  use.locfit = FALSE,
  fw = TRUE,
  rescale = TRUE,
  nxmargin = 15,
  nymargin = 100,
  a.nndefault = 0.3,
  ...
)
```

Arguments

x	Numerical vector or matrix: the conditioning variable(s).
y	Numerical vector: the response variable.
deg	Degree of local polynomial used in estimation.
link	Link function used in estimation. Default "identity". The other possibility is "log" which is recommended if degree > 0.
a	Optional bandwidth in x direction.
b	Optional bandwidth in y direction.
mean	Estimated mean of y x. If present, it will adjust conditional density to have this mean.
x.margin	Values in x-space on which conditional density is calculated. If not specified, an equi-spaced grid of nxmargin values over the range of x is used. If x is a matrix, x.margin should be a list of two numerical vectors.

<code>y.margin</code>	Values in y-space on which conditional density is calculated. If not specified, an equi-spaced grid of <code>nymargin</code> values over the range of <code>y</code> is used.
<code>x.name</code>	Optional name of <code>x</code> variable used in plots.
<code>y.name</code>	Optional name of <code>y</code> variable used in plots.
<code>use.locfit</code>	If TRUE, will use <code>locfit</code> for estimation. Otherwise <code>ksmooth</code> is used. <code>locfit</code> is used if <code>degree>0</code> or link not the identity or the dimension of <code>x</code> is greater than 1 even if <code>use.locfit=FALSE</code> .
<code>fw</code>	If TRUE (default), will use fixed window width estimation. Otherwise nearest neighbourhood estimation is used. If the dimension of <code>x</code> is greater than 1, nearest neighbourhood must be used.
<code>rescale</code>	If TRUE (default), will rescale the conditional densities to integrate to one.
<code>nxmargin</code>	Number of values used in <code>x.margin</code> by default.
<code>nymargin</code>	Number of values used in <code>y.margin</code> by default.
<code>a.nndefault</code>	Default nearest neighbour bandwidth (used only if <code>fw=FALSE</code> and <code>a</code> is missing.).
<code>...</code>	Additional arguments are passed to <code>locfit</code> .

Details

If bandwidths are omitted, they are computed using normal reference rules described in Bashtannyk and Hyndman (2001) and Hyndman and Yao (2002). Bias adjustment uses the method described in Hyndman, Bashtannyk and Grunwald (1996). If `deg>1` then estimation is based on the local parametric estimator of Hyndman and Yao (2002).

Value

A list with the following components:

<code>x</code>	grid in <code>x</code> direction on which density evaluated. Equal to <code>x.margin</code> if specified.
<code>y</code>	grid in <code>y</code> direction on which density is evaluated. Equal to <code>y.margin</code> if specified.
<code>z</code>	value of conditional density estimate returned as a matrix.
<code>a</code>	window width in <code>x</code> direction.
<code>b</code>	window width in <code>y</code> direction.
<code>x.name</code>	Name of <code>x</code> variable to be used in plots.
<code>y.name</code>	Name of <code>y</code> variable to be used in plots.

Author(s)

Rob J Hyndman

References

- Hyndman, R.J., Bashtannyk, D.M. and Grunwald, G.K. (1996) "Estimating and visualizing conditional densities". *Journal of Computational and Graphical Statistics*, **5**, 315-336.
- Bashtannyk, D.M., and Hyndman, R.J. (2001) "Bandwidth selection for kernel conditional density estimation". *Computational statistics and data analysis*, **36**(3), 279-298.
- Hyndman, R.J. and Yao, Q. (2002) "Nonparametric estimation and symmetry tests for conditional density functions". *Journal of Nonparametric Statistics*, **14**(3), 259-278.

See Also[cde.bandwidths](#)**Examples**

```
# Old faithful data
faithful.cde <- cde(faithful$waiting, faithful$eruptions,
  x.name="Waiting time", y.name="Duration time")
plot(faithful.cde)
plot(faithful.cde, plot.fn="hdr")

# Melbourne maximum temperatures with bias adjustment
x <- maxtemp[1:3649]
y <- maxtemp[2:3650]
maxtemp.cde <- cde(x, y,
  x.name="Today's max temperature", y.name="Tomorrow's max temperature")
# Assume linear mean
fit <- lm(y~x)
fit.mean <- list(x=6:45,y=fit$coef[1]+fit$coef[2]*(6:45))
maxtemp.cde2 <- cde(x, y, mean=fit.mean,
  x.name="Today's max temperature", y.name="Tomorrow's max temperature")
plot(maxtemp.cde)
```

cde.bandwidths

*Bandwidth calculation for conditional density estimation***Description**

Calculates bandwidths for kernel conditional density estimates. Methods described in Bashtannyk and Hyndman (2001) and Hyndman and Yao (2002).

Usage

```
cde.bandwidths(
  x,
  y,
  deg = 0,
  link = "identity",
  method = 1,
  y.margin,
  passes = 2,
  ngrid = 8,
  min.a = NULL,
  ny = 25,
  use.sample = FALSE,
  GCV = TRUE,
  b = NULL,
  ...
)
```

Arguments

x	Numerical vector: the conditioning variable.
y	Numerical vector: the response variable.
deg	Degree of local polynomial used in estimation.
link	Link function used in estimation. Default "identity". The other possibility is "log" which is recommended if degree > 0.
method	method = 1: Hyndman-Yao algorithm if deg>0; Bashtannyk-Hyndman algorithm if deg=0; method = 2: Normal reference rules; method = 3: Bashtannyk-Hyndman regression method if deg=0; method = 4: Bashtannyk-Hyndman bootstrap method if deg=0.
y.margin	Values in y-space on which conditional density is calculated. If not specified, an equi-spaced grid of 50 values over the range of y is used.
passes	Number of passes through Bashtannyk-Hyndman algorithm.
ngrid	Number of values of smoothing parameter in grid.
min.a	Smallest value of a to consider if method=1.
ny	Number of values to use for y margin if y.margin is missing.
use.sample	Used when regression method (3) is chosen.
GCV	Generalized cross-validation. Used only if method=1 and deg>0. If GCV=FALSE, method=1 and deg=0, then the AIC is used instead. The argument is ignored if deg=0 or method>1.
b	Value of b can be specified only if method=1 and deg>0. For deg=0 or method>1, this argument is ignored.
...	Other arguments control details for individual methods.

Details

Details of the various algorithms are in Bashtannyk and Hyndman (2001) and Hyndman and Yao (2002).

Value

a	Window width in x direction.
b	Window width in y direction.

Author(s)

Rob J Hyndman

References

- Hyndman, R.J., Bashtannyk, D.M. and Grunwald, G.K. (1996) "Estimating and visualizing conditional densities". *Journal of Computational and Graphical Statistics*, **5**, 315-336.
- Bashtannyk, D.M., and Hyndman, R.J. (2001) "Bandwidth selection for kernel conditional density estimation". *Computational statistics and data analysis*, **36**(3), 279-298.
- Hyndman, R.J. and Yao, Q. (2002) "Nonparametric estimation and symmetry tests for conditional density functions". *Journal of Nonparametric Statistics*, **14**(3), 259-278.

See Also

[cde](#)

Examples

```
bands <- cde.bandwidths(faithful$waiting,faithful$eruptions,method=2)
plot(cde(faithful$waiting,faithful$eruptions,a=bands$a,b=bands$b))
```

hdr

Highest Density Regions

Description

Calculates highest density regions in one dimension

Usage

```
hdr(
  x = NULL,
  prob = c(50, 95, 99),
  den = NULL,
  h = hdrbw(BoxCox(x, lambda), mean(prob)),
  lambda = 1,
  nn = 5000,
  all.modes = FALSE
)
```

Arguments

x	Numeric vector containing data. If x is missing then den must be provided, and the HDR is computed from the given density.
prob	Probability coverage required for HDRs
den	Density of data as list with components x and y. If omitted, the density is estimated from x using density .
h	Optional bandwidth for calculation of density.
lambda	Box-Cox transformation parameter where $0 \leq \lambda \leq 1$.
nn	Number of random numbers used in computing f-alpha quantiles.
all.modes	Return all local modes or just the global mode?

Details

Either `x` or `den` must be provided. When `x` is provided, the density is estimated using kernel density estimation. A Box-Cox transformation is used if `lambda!=1`, as described in Wand, Marron and Ruppert (1991). This allows the density estimate to be non-zero only on the positive real line. The default kernel bandwidth `h` is selected using the algorithm of Samworth and Wand (2010).

Hyndman's (1996) density quantile algorithm is used for calculation.

Value

A list of three components:

<code>hdr</code>	The endpoints of each interval in each HDR
<code>mode</code>	The estimated mode of the density.
<code>falpha</code>	The value of the density at the boundaries of each HDR.

Author(s)

Rob J Hyndman

References

- Hyndman, R.J. (1996) Computing and graphing highest density regions. *American Statistician*, **50**, 120-126.
- Samworth, R.J. and Wand, M.P. (2010). Asymptotics and optimal bandwidth selection for highest density region estimation. *The Annals of Statistics*, **38**, 1767-1792.
- Wand, M.P., Marron, J S., Ruppert, D. (1991) Transformations in density estimation. *Journal of the American Statistical Association*, **86**, 343-353.

See Also

[hdr.den](#), [hdr.boxplot](#)

Examples

```
# Old faithful eruption duration times
hdr(faithful$eruptions)
```

hdr.2d

Bivariate Highest Density Regions

Description

Calculates and plots highest density regions in two dimensions, including the bivariate HDR box-plot.

Usage

```

hdr.2d(
  x,
  y,
  prob = c(50, 95, 99),
  den = NULL,
  kde.package = c("ash", "ks"),
  h = NULL,
  xextend = 0.15,
  yextend = 0.15
)

hdr.boxplot.2d(
  x,
  y,
  prob = c(50, 99),
  kde.package = c("ash", "ks"),
  h = NULL,
  xextend = 0.15,
  yextend = 0.15,
  xlab = "",
  ylab = "",
  shadecols = "darkgray",
  pointcol = 1,
  outside.points = TRUE,
  ...
)

## S3 method for class 'hdr2d'
plot(
  x,
  shaded = TRUE,
  show.points = FALSE,
  outside.points = FALSE,
  pch = 20,
  shadecols = gray((length(x$alpha):1)/(length(x$alpha) + 1)),
  pointcol = 1,
  ...
)

```

Arguments

<code>x</code>	Numeric vector
<code>y</code>	Numeric vector of same length as <code>x</code> .
<code>prob</code>	Probability coverage required for HDRs
<code>den</code>	Bivariate density estimate (a list with elements <code>x</code> , <code>y</code> and <code>z</code> where <code>x</code> and <code>y</code> are grid values and <code>z</code> is a matrix of density values). If <code>NULL</code> , the density is estimated.

kde.package	Package to be used in calculating the kernel density estimate when den=NULL.
h	Pair of bandwidths passed to either ash2 or kde . If NULL, a reasonable default is used. Ignored if den is not NULL.
xextend	Proportion of range of x. The density is estimated on a grid extended by xextend beyond the range of x.
yextend	Proportion of range of y. The density is estimated on a grid extended by yextend beyond the range of y.
xlab	Label for x-axis.
ylab	Label for y-axis.
shadecols	Colors for shaded regions
pointcol	Color for outliers and mode
outside.points	If TRUE, the observations lying outside the largest HDR are shown.
...	Other arguments to be passed to plot.
shaded	If TRUE, the HDR contours are shown as shaded regions.
show.points	If TRUE, the observations are plotted over the top of the HDR contours.
pch	The plotting character used for observations.

Details

The density is estimated using kernel density estimation. Either [ash2](#) or [kde](#) is used to do the calculations. Then Hyndman's (1996) density quantile algorithm is used to compute the HDRs.

`hdr.2d` returns an object of class `hdr2d` containing all the information needed to compute the HDR contours. This object can be plotted using `plot.hdr2d`.

`hdr.boxplot.2d` produces a bivariate HDR boxplot. This is a special case of applying `plot.hdr2d` to an object computed using `hdr.2d`.

Value

Some information about the HDRs is returned. See code for details.

Author(s)

Rob J Hyndman

References

Hyndman, R.J. (1996) Computing and graphing highest density regions *American Statistician*, **50**, 120-126.

See Also

[hdr.boxplot](#)

Examples

```
x <- c(rnorm(200,0,1),rnorm(200,4,1))
y <- c(rnorm(200,0,1),rnorm(200,4,1))
hdr.boxplot.2d(x,y)

hdrinfo <- hdr.2d(x,y)
plot(hdrinfo, pointcol="red", show.points=TRUE, pch=3)
```

hdr.boxplot

*Highest Density Region Boxplots***Description**

Calculates and plots a univariate highest density regions boxplot.

Usage

```
hdr.boxplot(
  x,
  prob = c(99, 50),
  h = hdrbw(BoxCox(x, lambda), mean(prob)),
  lambda = 1,
  boxlabels = "",
  col = gray((9:1)/10),
  main = "",
  xlab = "",
  ylab = "",
  pch = 1,
  border = 1,
  outline = TRUE,
  space = 0.25,
  ...
)
```

Arguments

x	Numeric vector containing data or a list containing several vectors.
prob	Probability coverage required for HDRs density .
h	Optional bandwidth for calculation of density.
lambda	Box-Cox transformation parameter where $0 \leq \lambda \leq 1$.
boxlabels	Label for each box plotted.
col	Colours for regions of each box.
main	Overall title for the plot.
xlab	Label for x-axis.
ylab	Label for y-axis.

pch	Plotting character.
border	Width of border of box.
outline	If not <code>TRUE</code> , the outliers are not drawn.
space	The space between each box, between 0 and 0.5.
...	Other arguments passed to plot.

Details

The density is estimated using kernel density estimation. A Box-Cox transformation is used if $\lambda \neq 1$, as described in Wand, Marron and Ruppert (1991). This allows the density estimate to be non-zero only on the positive real line. The default kernel bandwidth h is selected using the algorithm of Samworth and Wand (2010).

Hyndman's (1996) density quantile algorithm is used for calculation.

Value

nothing.

Author(s)

Rob J Hyndman

References

- Hyndman, R.J. (1996) Computing and graphing highest density regions. *American Statistician*, **50**, 120-126.
- Samworth, R.J. and Wand, M.P. (2010). Asymptotics and optimal bandwidth selection for highest density region estimation. *The Annals of Statistics*, **38**, 1767-1792.
- Wand, M.P., Marron, J S., Ruppert, D. (1991) Transformations in density estimation. *Journal of the American Statistical Association*, **86**, 343-353.

See Also

[hdr.boxplot.2d](#), [hdr](#), [hdr.den](#)

Examples

```
# Old faithful eruption duration times
hdr.boxplot(faithful$eruptions)

# Simple bimodal example
x <- c(rnorm(100,0,1), rnorm(100,5,1))
par(mfrow=c(1,2))
boxplot(x)
hdr.boxplot(x)

# Highly skewed example
x <- exp(rnorm(100,0,1))
par(mfrow=c(1,2))
```

```
boxplot(x)
hdr.boxplot(x, lambda=0)
```

hdr.cde	<i>Calculate highest density regions continuously over some conditioned variable.</i>
---------	---

Description

Calculates and plots highest density regions for a conditional density estimate. Uses output from [cde](#).

Usage

```
hdr.cde(
  den,
  prob = c(50, 95, 99),
  plot = TRUE,
  plot.modes = TRUE,
  mden = rep(1, length(den$x)),
  threshold = 0.05,
  nn = 1000,
  xlim,
  ylim,
  xlab,
  ylab,
  border = TRUE,
  font = 1,
  cex = 1,
  ...
)
```

Arguments

den	Conditional density in the same format as the output from cde .
prob	Probability coverage level for HDRs
plot	Should HDRs be plotted? If FALSE, results are returned.
plot.modes	Should modes be plotted as well as HDRs?
mden	Marginal density in the x direction. When small, the HDRs won't be plotted. Default is uniform so all HDRs are plotted.
threshold	Threshold for margin density. HDRs are not plotted if the margin density mden is lower than this value.
nn	Number of points to be sampled from each density when estimating the HDRs.
xlim	Limits for x-axis.

ylim	Limits for y-axis.
xlab	Label for x-axis.
ylab	Label for y-axis.
border	Show border of polygons
font	Font to be used in plot.
cex	Size of characters.
...	Other arguments passed to plotting functions.

Value

hdr	array (a,b,c) where a specifies conditioning value, b gives the HDR end-points and c gives the probability coverage.
modes	estimated mode of each conditional density

Author(s)

Rob J Hyndman

References

Hyndman, R.J., Bashtannyk, D.M. and Grunwald, G.K. (1996) "Estimating and visualizing conditional densities". *Journal of Computational and Graphical Statistics*, **5**, 315-336.

See Also

[cde](#), [hdr](#)

Examples

```
faithful.cde <- cde(faithful$waiting,faithful$eruptions)
plot(faithful.cde,xlab="Waiting time",ylab="Duration time",plot.fn="hdr")
```

hdr.den

Density plot with Highest Density Regions

Description

Plots univariate density with highest density regions displayed

Usage

```
hdr.den(
  x,
  prob = c(50, 95, 99),
  den,
  h = hdrbw(BoxCox(x, lambda), mean(prob)),
  lambda = 1,
  xlab = NULL,
  ylab = "Density",
  ylim = NULL,
  plot.lines = TRUE,
  col = 2:8,
  bgcol = "gray",
  legend = FALSE,
  ...
)
```

Arguments

x	Numeric vector containing data. If x is missing then den must be provided, and the HDR is computed from the given density.
prob	Probability coverage required for HDRs
den	Density of data as list with components x and y. If omitted, the density is estimated from x using density .
h	Optional bandwidth for calculation of density.
lambda	Box-Cox transformation parameter where $0 \leq \lambda \leq 1$.
xlab	Label for x-axis.
ylab	Label for y-axis.
ylim	Limits for y-axis.
plot.lines	If TRUE, will show how the HDRs are determined using lines.
col	Colours for regions.
bgcol	Colours for the background behind the boxes. Default "gray", if NULL no box is drawn.
legend	If TRUE add a legend on the right of the boxes.
...	Other arguments passed to plot.

Details

Either x or den must be provided. When x is provided, the density is estimated using kernel density estimation. A Box-Cox transformation is used if $\lambda \neq 1$, as described in Wand, Marron and Ruppert (1991). This allows the density estimate to be non-zero only on the positive real line. The default kernel bandwidth h is selected using the algorithm of Samworth and Wand (2010).

Hyndman's (1996) density quantile algorithm is used for calculation.

Value

a list of three components:

hdr	The endpoints of each interval in each HDR
mode	The estimated mode of the density.
falpha	The value of the density at the boundaries of each HDR.

Author(s)

Rob J Hyndman

References

- Hyndman, R.J. (1996) Computing and graphing highest density regions. *American Statistician*, **50**, 120-126.
- Samworth, R.J. and Wand, M.P. (2010). Asymptotics and optimal bandwidth selection for highest density region estimation. *The Annals of Statistics*, **38**, 1767-1792.
- Wand, M.P., Marron, J S., Ruppert, D. (1991) Transformations in density estimation. *Journal of the American Statistical Association*, **86**, 343-353.

See Also

[hdr](#), [hdr.boxplot](#)

Examples

```
# Old faithful eruption duration times
hdr.den(faithful$eruptions)

# Simple bimodal example
x <- c(rnorm(100,0,1), rnorm(100,5,1))
hdr.den(x)
```

hdrbw

Highest Density Region Bandwidth

Description

Estimates the optimal bandwidth for 1-dimensional highest density regions

Usage

```
hdrbw(x, HDRlevel, gridsize = 801, nMChdr = 1e+06, graphProgress = FALSE)
```

Arguments

x	Numerical vector containing data.
HDRlevel	HDR-level as defined in Hyndman (1996). Setting ‘HDRlevel’ equal to p ($0 < p < 1$) corresponds to a probability of $1-p$ of inclusion in the highest density region.
gridsize	the number of equally spaced points used for binned kernel density estimation.
nMChdr	the size of the Monte Carlo sample used for density quantile approximation of the highest density region, as described in Hyndman (1996).
graphProgress	logical flag: if ‘TRUE’ then plots showing the progress of the bandwidth selection algorithm are produced.

Details

This is a plug-in rule for bandwidth selection tailored to highest density region estimation

Value

A numerical vector of length 1.

Author(s)

Matt Wand

References

- Hyndman, R.J. (1996). Computing and graphing highest density regions. *The American Statistician*, **50**, 120-126.
- Samworth, R.J. and Wand, M.P. (2010). Asymptotics and optimal bandwidth selection for highest density region estimation. *The Annals of Statistics*, **38**, 1767-1792.

Examples

```
HDRlevelVal <- 0.55
x <- faithful$eruptions
hHDR <- hdrbw(x, HDRlevelVal)
HDRhat <- hdr.den(x, prob=100*(1-HDRlevelVal), h=hHDR)
```

hdrconf

HDRs with confidence intervals

Description

Calculates Highest Density Regions with confidence intervals.

Usage

```
hdrconf(x, den, prob = 95, conf = 95)
```

Arguments

x	Numeric vector containing data.
den	Density of data as list with components x and y.
prob	Probability coverage for HDRs.
conf	Confidence for limits on HDR.

Value

hdrconf returns list containing the following components:

hdr	Highest density regions
hdr.lo	Highest density regions corresponding to lower confidence limit.
hdr.hi	Highest density regions corresponding to upper confidence limit.
falpha	Values of f_α corresponding to HDRs.
falpha.ci	Values of f_α corresponding to lower and upper limits.

Author(s)

Rob J Hyndman

References

Hyndman, R.J. (1996) Computing and graphing highest density regions *American Statistician*, **50**, 120-126.

See Also

[hdr](#), [plot.hdrconf](#)

Examples

```
x <- c(rnorm(100,0,1),rnorm(100,4,1))
den <- density(x,bw=hdrbw(x,50))
trueden <- den
trueden$y <- 0.5*(exp(-0.5*(den$x*den$x)) + exp(-0.5*(den$x-4)^2))/sqrt(2*pi)
sortx <- sort(x)

par(mfcol=c(2,2))
for(conf in c(50,95))
{
  m <- hdrconf(sortx,trueden,conf=conf)
  plot(m,trueden,main=paste(conf,"% HDR from true density"))
  m <- hdrconf(sortx,den,conf=conf)
  plot(m,den,main=paste(conf,"% HDR from empirical density\n(n=200)"))
}
```

`hdrscatterplot`*Scatterplot showing bivariate highest density regions*

Description

Produces a scatterplot where the points are coloured according to the bivariate HDRs in which they fall.

Usage

```
hdrscatterplot(  
  x,  
  y,  
  levels = c(1, 50, 99),  
  kde.package = c("ash", "ks"),  
  noutliers = NULL,  
  label = NULL  
)
```

Arguments

<code>x</code>	Numeric vector or matrix with 2 columns.
<code>y</code>	Numeric vector of same length as <code>x</code> .
<code>levels</code>	Percentage coverage for HDRs
<code>kde.package</code>	Package to be used in calculating the kernel density estimate when <code>den=NULL</code> .
<code>noutliers</code>	Number of outliers to be labelled. By default, all points outside the largest HDR are labelled.
<code>label</code>	Label of outliers of same length as <code>x</code> and <code>y</code> . By default, all outliers are labelled as the row index of the point (<code>x</code> , <code>y</code>).

Details

The bivariate density is estimated using kernel density estimation. Either [ash2](#) or [kde](#) is used to do the calculations. Then Hyndman's (1996) density quantile algorithm is used to compute the HDRs. The scatterplot of (x,y) is created where the points are coloured according to which HDR they fall. A `ggplot` object is returned.

Author(s)

Rob J Hyndman

See Also

[hdr.boxplot.2d](#)

Examples

```
x <- c(rnorm(200, 0, 1), rnorm(200, 4, 1))
y <- c(rnorm(200, 0, 1), rnorm(200, 4, 1))
hdrscatterplot(x, y)
hdrscatterplot(x, y, label = paste0("p", 1:length(x)))
```

lane2

Speed-Flow data for Californian Freeway

Description

These are two data sets collected in 1993 on two individual lanes (lane 2 and lane 3) of the 4-lane Californian freeway I-880. The data were collected by loop detectors, and the time units are 30 seconds per observation (see Petty et al., 1996, for details).

Usage

```
lane2; lane3
```

Format

Two data frames (lane2 and lane3) each with 1318 observations on the following two variables:

flow a numeric vector giving the traffic flow in vehicles per lane per hour.

speed a numeric vector giving the speed in miles per hour.

Details

The data is examined in Einbeck and Tutz (2006), using a nonparametric approach to multi-valued regression based on conditional mean shift.

Source

Petty, K.F., Noeimi, H., Sanwal, K., Rydzewski, D., Skabardonis, A., Varaiya, P., and Al-Deek, H. (1996). "The Freeway Service Patrol Evaluation Project: Database Support Programs, and Accessibility". *Transportation Research Part C: Emerging Technologies*, **4**, 71-85.

The data is provided by courtesy of CALIFORNIA PATH, Institute of Transportation Studies, University of California, Berkeley.

References

Einbeck, J., and Tutz, G. (2006). "Modelling beyond regression functions: an application of multimodal regression to speed-flow data". *Journal of the Royal Statistical Society, Series C (Applied Statistics)*, **55**, 461-475.

Examples

```
plot(lane2)
plot(lane3)
```

maxtemp	<i>Daily maximum temperatures in Melbourne, Australia</i>
---------	---

Description

Daily maximum temperatures in Melbourne, Australia, from 1981-1990. Leap days have been omitted.

Usage

```
maxtemp
```

Format

Time series of frequency 365.

Source

Hyndman, R.J., Bashtannyk, D.M. and Grunwald, G.K. (1996) "Estimating and visualizing conditional densities". *Journal of Computational and Graphical Statistics*, **5**, 315-336.

Examples

```
plot(maxtemp)
```

modalreg	<i>Nonparametric Multimodal Regression</i>
----------	--

Description

Nonparametric multi-valued regression based on the modes of conditional density estimates.

Usage

```
modalreg(  
  x,  
  y,  
  xfix = seq(min(x), max(x), l = 50),  
  a,  
  b,  
  deg = 0,  
  iter = 30,  
  P = 2,  
  start = "e",  
  prun = TRUE,  
  prun.const = 10,
```

```

plot.type = c("p", 1),
labels = c("", "x", "y"),
pch = 20,
...
)

```

Arguments

<code>x</code>	Numerical vector: the conditioning variable.
<code>y</code>	Numerical vector: the response variable.
<code>xfix</code>	Numerical vector corresponding to the input values of which the fitted values shall be calculated.
<code>a</code>	Optional bandwidth in x -direction.
<code>b</code>	Optional bandwidth in y -direction.
<code>deg</code>	Degree of local polynomial used in estimation (0 or 1).
<code>iter</code>	Positive integer giving the number of mean shift iterations per point and branch.
<code>P</code>	Maximal number of branches.
<code>start</code>	Character determining how the starting points are selected. "q": proportional to quantiles; "e": equidistant; "r": random. All, "q", "e", and "r", give starting points which are constant over x . As an alternative, the choice "v" gives variable starting points, which are equal to "q" for the smallest x , and equal to the previously fitted values for all subsequent x .
<code>prun</code>	Boolean. If TRUE, parts of branches are dismissed (in the plotted output) where their associated kernel density value falls below the threshold $1/(\text{prun.const} * (\max(x) - \min(x)) * (\max(y) - \min(y)))$.
<code>prun.const</code>	Numerical value giving the constant used above (the higher, the less pruning)
<code>plot.type</code>	Vector with two elements. The first one is character-valued, with possible values "p", "l", and "n". If equal to "n", no plotted output is given at all. If equal to "p", fitted curves are symbolized as points in the graphical output, otherwise as lines. The second vector component is a numerical value either being 0 or 1. If 1, the position of the starting points is depicted in the plot, otherwise omitted.
<code>labels</code>	Vector of three character strings. The first one is the "main" title of the graphical output, the second one is the label of the x axis, and the third one the label of the y axis.
<code>pch</code>	Plotting character. The default corresponds to small bullets.
<code>...</code>	Other arguments passed to <code>cde.bandwidths</code> .

Details

Computes multi-modal nonparametric regression curves based on the maxima of conditional density estimates. The tool for the estimation is the conditional mean shift as outlined in Einbeck and Tutz (2006). Estimates of the conditional modes might fluctuate highly if `deg=1`. Hence, `deg=0` is recommended. For bandwidth selection, the hybrid rule introduced by Bashtannyk and Hyndman (2001) is employed if `deg=0`. This corresponds to the setting `method=1` in function `cde.bandwidths`. For `deg=1` automatic bandwidth selection is not supported.

Value

A list with the following components:

<code>xfix</code>	Grid of predictor values at which the fitted values are calculated.
<code>fitted.values</code>	A $[P \times \text{length}(\text{xfix})]$ - matrix with fitted j -th branch in the j -th row ($1 \leq j \leq P$)
<code>bandwidths</code>	A vector with bandwidths a and b .
<code>density</code>	A $[P \times \text{length}(\text{xfix})]$ - matrix with estimated kernel densities. This will only be computed if <code>prun=TRUE</code> .
<code>threshold</code>	The pruning threshold.

Author(s)

Jochen Einbeck (2007)

References

- Einbeck, J., and Tutz, G. (2006) "Modelling beyond regression functions: an application of multimodal regression to speed-flow data". *Journal of the Royal Statistical Society, Series C (Applied Statistics)*, **55**, 461-475.
- Bashtannyk, D.M., and Hyndman, R.J. (2001) "Bandwidth selection for kernel conditional density estimation". *Computational Statistics and Data Analysis*, **36**(3), 279-298.

See Also

[cde.bandwidths](#)

Examples

```
lane2.fit <- modalreg(lane2$flow, lane2$speed, xfix=(1:55)*40, a=100, b=4)
```

plot.cde

Plots conditional densities

Description

Produces stacked density plots or highest density region plots for a univariate density conditional on one covariate.

Usage

```
## S3 method for class 'cde'
plot(
  x,
  firstvar = 1,
  mfrow = n2mfrow(dim(x$z)[firstvar]),
  plot.fn = "stacked",
  x.name,
  margin = NULL,
  ...
)
```

Arguments

<code>x</code>	Output from cde .
<code>firstvar</code>	If there is more than one conditioning variable, <code>firstvar</code> specifies which variable to fix first.
<code>mfrow</code>	If there is more than one conditioning variable, <code>mfrow</code> is passed to par before plotting.
<code>plot.fn</code>	Specifies which plotting function to use: "stacked" results in stacked conditional densities and "hdr" results in highest density regions.
<code>x.name</code>	Name of <code>x</code> (conditioning) variable for use on x-axis.
<code>margin</code>	Marginal density of conditioning variable. If present, only conditional densities corresponding to non-negligible marginal densities will be plotted.
<code>...</code>	Additional arguments to plot.

Value

If `plot.fn=="stacked"` and there is only one conditioning variable, the function returns the output from [persp](#). If `plot.fn=="hdr"` and there is only one conditioning variable, the function returns the output from [hdr.cde](#). When there is more than one conditioning variable, nothing is returned.

Author(s)

Rob J Hyndman

References

Hyndman, R.J., Bashtannyk, D.M. and Grunwald, G.K. (1996) "Estimating and visualizing conditional densities". *Journal of Computational and Graphical Statistics*, **5**, 315-336.

See Also

[hdr.cde](#), [cde](#), [hdr](#)

Examples

```
faithful.cde <- cde(faithful$waiting,faithful$eruptions,
  x.name="Waiting time", y.name="Duration time")
plot(faithful.cde)
plot(faithful.cde,plot.fn="hdr")
```

plot.hdrconf	<i>Plot HDRs with confidence intervals</i>
--------------	--

Description

Plots Highest Density Regions with confidence intervals.

Usage

```
## S3 method for class 'hdrconf'
plot(x, den, ...)
```

Arguments

x	Output from hdrconf.
den	Density of data as list with components x and y.
...	Other arguments are passed to plot.

Value

None

Author(s)

Rob J Hyndman

References

Hyndman, R.J. (1996) Computing and graphing highest density regions *American Statistician*, **50**, 120-126.

See Also

[hdrconf](#)

Examples

```
x <- c(rnorm(100,0,1),rnorm(100,4,1))
den <- density(x,bw=bw.SJ(x))
trueden <- den
trueden$y <- 0.5*(exp(-0.5*(den$x*den$x)) + exp(-0.5*(den$x-4)^2))/sqrt(2*pi)
sortx <- sort(x)

par(mfcol=c(2,2))
for(conf in c(50,95))
{
  m <- hdrconf(sortx,trueden,conf=conf)
  plot(m,trueden,main=paste(conf,"% HDR from true density"))
  m <- hdrconf(sortx,den,conf=conf)
  plot(m,den,main=paste(conf,"% HDR from empirical density\n(n=200)"))
}
```

shades*Shades*

Description

A simple function to generate shade of one color by changing its opacity

Usage

```
shades(color, n)
```

Arguments

color	the name or idea of a R color
n	number of shades wanted

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