Package 'sim2Dpredictr'

July 23, 2025

Title Simulate Outcomes Using Spatially Dependent Design Matrices

Version 0.1.1

Description Provides tools for simulating spatially dependent predictors (continuous or binary), which are used to generate scalar outcomes in a (generalized) linear model framework. Continuous predictors are generated using traditional multivariate normal distributions or Gauss Markov random

fields with several correlation function approaches (e.g., see Rue (2001) <doi:10.1111/1467-9868.00288>

and Furrer and Sain (2010) <doi:10.18637/jss.v036.i10>), while binary predictors are generated using

a Boolean model (see Cressie and Wikle (2011, ISBN: 978-0-471-69274-4)). Parameter vectors exhibiting spatial clustering can also be easily specified by the user.

Depends R (>= 3.5.0)

License GPL-3

Encoding UTF-8

Imports MASS, Rdpack, spam (>= 2.2-0), tibble, dplyr, matrixcalc

RdMacros Rdpack

RoxygenNote 7.2.3

Suggests knitr, rmarkdown, testthat, V8

URL https://github.com/jmleach-bst/sim2Dpredictr

BugReports https://github.com/jmleach-bst/sim2Dpredictr

NeedsCompilation no

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Repository CRAN

Date/Publication 2023-04-03 13:00:02 UTC

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beta_builder

Create a Parameter Vector from Lattice Locations

Description

Specify the locations in the lattice/image that have non-zero parameters as well as the values for those parameters, and the function creates the parameter vector that matches the correct locations in the design matrix.

Usage

```
beta_builder(
  row.index,
  col.index,
  im.res,
  B0 = 0,
  B.values,
  index.type = "manual",
  decay.fn = "gaussian",
  phi = 0.5,
  max.d = Inf,
  h,
  w,
  bayesian = FALSE,
```

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```
bayesian.dist = NULL,
  bayesian.scale = NULL,
  output.indices = TRUE
)
```

Arguments

row.index, col.index

Vectors of row/columns indices for non-zero parameters. If index.type = "manual", each vector should contain specific coordinates. If index.type = "rectangle", each vector should specify rectangle length; e.g. row.index = 1:3 means the rectangle's 'length' is from rows 1 to 3. If index.type = "ellipse", the arguments should be scalar values specifying the row/column coordinates for the center of the ellipse. If index.type = "decay", the arguments should specify the row/column coordinates, respectively, of the peak parameter value.

im.res

A vector specifying the dimension/resolution of the image. The first entry is the number of 'rows' in the lattice/image, and the second entry is the number of 'columns' in the lattice/image.

B0

is the "true" intercept value; default is 0.

B.values

is a vector "true" parameter values for non-zero parameters. The order of assignment is by row. If B.values argument is a single value, then all non-zero parameters are assigned to that value, unless decay. fn has been specified, in which case B. values is the "peak", and non-zero parameters decay smoothly by distance.

index.type

is one of index.type = c("manual", "rectangle", "ellipse", "decay")

- index.type = "manual" uses row.index and col.index arguments to specify manually selected non-zero locations. This setting is good for irregular shaped regions.
- index.type = "rectangle" uses row.index and col.index arguments to specify a rectangular region of non-zero parameters.
- index.type = "ellipse" uses w and h arguments to specify elliptical region of non-zero parameters.
- index.type = "decay" allows the user to specify a peak location with row.index and col.index, as with index.type = "ellipse". However, the non-zero parameter values decay as a function of distance from the

decay.fn

An argument to specify the decay function of non-zero parameters decay from the peak when index.type = "decay". Options are "exponential" or "gaussian". The rate of decay is given by B.values * exp(-phi * d) and B.values * $exp(-phi*d^2)$ for "exponential" and "gaussian", respectively. The default is decay. fn = "gaussian". Note that d is the Euclidean distance between the peak and a specified location, while phi is the rate of decay and is set by the user with phi.

phi

A scalar value greater than 0 that determines the decay rate of non-zero parameters when index.type = "decay". The default is phi = 0.5.

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When index.type = "decay", max.d determines the maximum Euclidean dismax.d tance from the peak that is allowed to be non-zero; parameters for locations further than max.d from the peak are set to zero. If this argument is not set by the user then all parameter values are determined by the decay function. w, h If index.type = "ellipse" then the width and height of the ellipse, respectively. If TRUE, then parameters are drawn from distributions based on initial B. values bayesian vector. Default is FALSE. bayesian.dist When bayesian = TRUE, specifies the distribution of the parameters. Options are "gaussian" and uniform. bayesian.scale A list. When bayesian = TRUE and bayesian.dist = "gaussian", specifies the sd for the distributions of parameters. When bayesian = TRUE and bayesian.dist = "uniform", specifies the width for the uniform distributions for the parameters. The first entry should be one of "unique", "binary". If "unique", then the second entry in the list should be a vector with length equal to B. values + 1 with unique values for the sd's/widths, including B0. B0 can be set to a constant value by setting the first position of bayesian.scale[[2]] to 0. If "binary", then the second entry in the list should be a 3-element vector whose first entry is the sd/width of B0, second entry the sd/width of "non-zero" or "important" parameters, and the third entry is the sd/width of the "zero" or "irrelevant" parameters. output.indices If output.indices = TRUE, then the first element of the returned list contains the indices for the non-zero parameter locations (Default). If output.indices

Value

A 2-element list containing (1) indices for the locations of "true" non-zero parameters, and (2) a parameter vector.

= FALSE, then only the parameter vector is returned.

Note

The order of the parameters is by row. That is, if the lattice/image is 4x4, then parameters 1-4 make up the first row, 5-8 then second, and so forth.

```
## In this example, I = 10, J = 10; Thus:
## (5, 5) \rightarrow 10 * (5 - 1) + 5 = 45
## (5, 6) \rightarrow 10 * (5 - 1) + 6 = 46
## (6, 5) \rightarrow 10 * (6 - 1) + 5 = 55
## (6, 6) \rightarrow 10 * (6 - 1) + 6 = 45
## length 101 (includes B0 w/ 100 variable parameter values)
length(Bex1$B)
## example: index.type = "rectangle"
Bex2 <- beta_builder(row.index = 12:15, col.index = 6:19,</pre>
                     im.res = c(20, 20), B0 = 16,
                     B.values = 1:(length(12:15) * length(6:19)),
                     index.type = "rectangle")
Bex2
matrix(Bex2$B[-1], nrow = 20, byrow = TRUE)
## example: index.type = "ellipse"
Bex3 <- beta_builder(row.index = 4, col.index = 5,</pre>
                     im.res = c(10, 10),
                     B0 = 16, B. values = 3,
                     index.type = "ellipse",
                     h = 5, w = 4
Bex3
matrix(Bex3\$B[-1], nrow = 10, byrow = TRUE)
## decaying parameter values
Bex4 <- beta_builder(row.index = 10, col.index = 20,</pre>
                      im.res = c(30, 30), B0 = 0, B.values = 10,
                      index.type = "decay", \max.d = 7,
                      output.indices = FALSE)
inf_2D_image(B = Bex4, im.res = c(30, 30), binarize.B = FALSE)
Bex5 <- beta_builder(row.index = 4, col.index = 5,</pre>
                      im.res = c(10, 10),
                      B0 = 16, B. values = 5,
                      index.type = "ellipse",
                      h = 5, w = 4,
                      bayesian = TRUE,
                      bayesian.dist = "gaussian",
                      bayesian.scale = list("binary", c(0, 1, 0.25)))
inf_2D_image(B = Bex5$B, im.res = c(10, 10), binarize.B = FALSE)
```

Description

The function first builds a correlation matrix with correlation.builder, converts that matrix to a covariance matrix if necessary, and then takes the Cholesky decomposition of the matrix using either base R or the R package spam. Note that spam is particularly effective when the matrix is sparse.

Usage

```
chol_s2Dp(
 matrix.type = "cov",
  im.res,
 use.spam = FALSE,
  corr.structure = "ar1",
  rho = NULL,
  phi = NULL,
  tau = 1,
  alpha = 0.75,
  corr.min = NULL,
  neighborhood = "none",
 w = NULL,
  h = NULL,
  r = NULL
  print.R = FALSE,
  print.S = FALSE,
  print.Q = FALSE,
  sigma = 1,
  triangle = "upper",
  print.all = FALSE,
  round.d = FALSE,
  return.cov = TRUE,
  return.prec = TRUE
)
```

Arguments

matrix.type	Determines whether to build a covariance matrix, "cov", or a precision matrix, "prec". See correlation_builder{sim2Dpredictr} and precision_builder{sim2Dpredictr} for more details.
im.res	A vector defining the dimension of spatial data. The first entry is the number of rows and the second entry is the number of columns.
use.spam	If use.spam = TRUE then use tools from the R package spam; otherwise, base R functions are employed. For large dimension MVN with sparse correlation structure, spam is recommended; otherwise, base R may be faster. Defaults to FALSE.
corr.structure	One of "ar1", exponential, gaussian, or "CS". Correlations between locations i and j are rho ^d for corr.structure = "ar1", $exp(-phi*d)$ for corr.structure = "exponential", $exp(-phi*d)$ for corr.structure = "gaussian", and

rho when corr.structure = "CS". Note that d is the Euclidean distance between locations i and j.

rho This is the maximum possible correlation between locations i and j. For all i,j rho MUST be between -1 and 1.

A scalar value greater than 0 that determines the decay rate of correlation. This argument is only utilized when corr.structure %in% c("exponential",

"gaussian").

A vector containing precision parameters. If of length 1, then all precisions are assumed equal. Otherwise the length of tau should equal the number of

A scalar value between 0 and 1 that defines the strength of correlations. Note that when alpha = 0 the data are independent and when alpha = 1, the joint distribution is the improper Intrinsic Autoregression (IAR), which cannot be used to generate data. Note also that while alpha does control dependence it is not interpretable as a correlation.

Scalar value to specify the minimum non-zero correlation. Any correlations below corr.min are set to 0. Especially for high image resolution using this option can result in a sparser covariance matrix, which may significantly speed up draws when using spam. This option is preferred to using neighborhood and associated arguments when the primary concern is to avoid very small correlations and improve computation efficiency. Default is NULL, which places no restrictions on the correlations.

Defines the neighborhood within which marginal correlations are non-zero. The default is "none", which allows marginal correlations to extend indefinitely. neighborhood = "round" defines a circular neighborhood about locations and neighborhood = "rectangle" defines a rectangular neighborhood about locations. Note that this argument differs from that in precision_builder, in which neighborhood defines conditional non-zero correlations.

If neighborhood = "rectangle" then w and h are the number of locations to the left/right and above/below a location i that define its neighborhood. Any locations outside this neighborhood have have zero correlation with location i.

If neighborhood = "round", then if locations i,j are separated by distance $d \ge r$, the correlation between them is zero.

print.R, print.S, print.Q

Logical. When TRUE, then print the correlation, covariance, or precision matrix before taking the Cholesky decomposition. If sigma = 1, then S = R.

Specify the desired standard deviations; the default is 1, in which case the Cholesky decomposition is of a correlation matrix. If sigma != 1, then the Cholesky decomposition is of a covariance Matrix.

- If sigma is a vector then length(sigma) must be equal to the total number of locations, i.e. (n.row * n.col)by(n.row * n.col).
- sigma can take any scalar value when specifying common standard deviation.

Determine whether to output an upper (triangle = "upper") or lower (triangle = "lower") triangular matrix.

tau

phi

alpha

corr.min

neighborhood

w, h

.

sigma

triangle

print.all If print.all = TRUE, then prints each correlation and allows you to check whether the correlations are as you intended. This option is NOT recommended for large point lattices/images.

round.d If round.d = TRUE, then d is rounded to the nearest whole number. return.cov, return.prec

Logical. When TRUE, also return the covariance or precision matrix, respectively. This is recommended when using spam to generate draws from the MVN.

Value

Matrix of dimension (n.row x n.col) x (n.row x n.col). If either return.cov or return.prec is TRUE, then returns a list where the first element is the covariance or precision matrix, and the second element is the Cholesky factor.

References

Banerjee S, Carlin BP, Gelfand AE (2015). *Hierarchical Modeling and Analysis for Spatial Data*, Second edition. Chapman & Hall/CRC, Boca Raton, Florida.

Ripley BD (1987). Stochastic Simulation. John Wiley & Sons. doi:10.1002/9780470316726.

Rue H (2001). "Fast Sampling of Gaussian Markov Random Fields." *Journal of the Royal Statistical Society B*, **63**, 325-338. doi:10.1111/14679868.00288.

Furrer R, Sain SR (2010). "spam: A Sparse Matrix R Package with Emphasis on MCMC Methods for Gaussian Markov Random Fields." *Journal of Statistical Software*, **36**(10), 1-25. https://www.jstatsoft.org/v36/i10/.

```
## Use R package spam for Cholesky decomposition
R_spam <- chol_s2Dp(im.res = c(3, 3), matrix.type = "prec",</pre>
                     use.spam = TRUE, neighborhood = "ar1",
                     triangle = "upper")
## Use base R for Cholesky decomposition
R_base <- chol_s2Dp(corr.structure = "ar1",</pre>
                     im.res = c(3, 3), rho = 0.15,
                     neighborhood = "round",
                     r = 3, use.spam = FALSE)
## Specify standard deviations instead of default of sigma = 1.
R_sd <- chol_s2Dp(corr.structure = "ar1",</pre>
                   im.res = c(3, 3), rho = 0.15,
                   neighborhood = "round", r = 3,
                   sigma = runif(9, 1.1, 4))
## Not run:
## Print options ON
R_pr_on <- chol_s2Dp(corr.structure = "ar1",</pre>
                      im.res = c(3, 3), rho = 0.15,
                      sigma = 1:9, neighborhood = "round",
                      r = 3, print.R = TRUE, print.S = TRUE)
```

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```
## End(Not run)
```

classify_multiclass

Classify subjects based on predicted probabilities for each class

Description

Classify subjects based on predicted probabilities for each class. The predicted probabilities can be input by the user or calculated within function using parameter estimates and design matrix for a multinomial regression model.

Usage

```
classify_multiclass(
  predicted.probs = NULL,
  category.names,
  keep.probs = TRUE,
  B = NULL
  X = NULL
  X.incl.X0 = FALSE
)
```

Arguments

predicted.probs

A matrix where the number of rows is equal to the number of subjects and the number of columns equals the number of categories. predicted.probs[i, j] contains the probability that subject i belongs to category j.

category.names A vector containing the names of each category. The order of names should match the order of columns in predicted probs; correspondingly, the length of the vector should equal the number of columns in predicted. probs.

keep.probs

Logical. When TRUE, the output is data frame consisting of the information in predicted.probs with an additional column predicted.class that contains the predicted class for each subject. When FALSE, a vector of the predicted classes is returned.

В

A list, each element of which contains a parameter vector. The list should have length V - 1, i.e., should contain parameter values associated with all categories except the reference category, following Agresti (2007). Alternatively, B may be a list of length V if one desires to specify parameters for every category, i.e., the over-parameterized model used in Friedman (2010).

Χ

A matrix, each row of which contains subject covariate/predictor values.

X.incl.X0

Logical. When TRUE, X should contain column of 1's for the intercept. Otherwise, a column of 1's is generated internally. Default is FALSE.

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Details

Classification for each subject is determined based on the category with highest predicted probability.

Value

Depending on the option selected for keep.probs, returns a data frame or vector.

Examples

```
## number of categories
vt <- 3
## covariate values
xt \leftarrow matrix(rnorm(10 * 2), ncol = 2, nrow = 10)
## list of parameter vectors (over-parameterized model)
bu <- list(b1 = c(0, 0.25, 0.25),
           b2 = c(0, -0.25, -0.25),
           b3 = c(0, 0.25, -0.25))
## subject specific probabilities for each category
## (over-parameterized model)
prp <- generate_multinom_probs(V = vt, X = xt, B = bu)</pre>
classify_multiclass(predicted.probs = prp,
                    category.names = c("A", "B", "C"))
## generate predicted probabilities within function
classify_multiclass(predicted.probs = NULL,
                    category.names = c("A", "B", "C"),
                    X = xt, B = bu
```

correlation_builder Build a Correlation Matrix for 2D Spatial Data

Description

This function "builds" a correlation matrix based on user specifications.

Usage

```
correlation_builder(
  corr.structure = "ar1",
  im.res,
  corr.min = NULL,
  neighborhood = "none",
  rho = NULL,
```

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```
phi = NULL,
w = NULL,
h = NULL,
r = NULL,
print.all = FALSE,
round.d = FALSE)
```

Arguments

corr.structure One of "ar1", exponential, gaussian, or "CS". Correlations between loca-

tions i and j are rho^d for corr. structure = "ar1", exp(-phi*d) for corr. structure

= "exponential", $exp(-phi*d^2)$ for corr.structure = "gaussian", and rho when corr.structure = "CS". Note that d is the Euclidean distance be-

im.res A vector defining the dimension of spatial data. The first entry is the number of rows and the second entry is the number of columns.

corr.min Scalar value to specify the minimum non-zero correlation. Any correlations

below corr.min are set to 0. Especially for high image resolution using this option can result in a sparser covariance matrix, which may significantly speed up draws when using spam. This option is preferred to using neighborhood and associated arguments when the primary concern is to avoid very small correlations and improve computation efficiency. Default is NULL, which places no

restrictions on the correlations.

tween locations i and j.

neighborhood Defines the neighborhood within which marginal correlations are non-zero. The

default is "none", which allows marginal correlations to extend indefinitely. neighborhood = "round" defines a circular neighborhood about locations and neighborhood = "rectangle" defines a rectangular neighborhood about locations. Note that this argument differs from that in precision_builder, in

which neighborhood defines conditional non-zero correlations.

rho This is the maximum possible correlation between locations i and j. For all i,j

rho MUST be between -1 and 1.

phi A scalar value greater than 0 that determines the decay rate of correlation.

This argument is only utilized when corr.structure %in% c("exponential",

"gaussian").

w, h If neighborhood = "rectangle" then w and h are the number of locations to the left/right and above/below a location i that define its neighborhood. Any

locations outside this neighborhood have have zero correlation with location i.

r If neighborhood = "round", then if locations i,j are separated by distance $d \geq$

r, the correlation between them is zero.

print.all If print.all = TRUE, then prints each correlation and allows you to check whether

the correlations are as you intended. This option is NOT recommended for large

point lattices/images.

round.d If round.d = TRUE, then d is rounded to the nearest whole number.

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Value

Returns (nr * nc)by(nr * nc) correlation matrix.

Note

Caution is recommended when using corr.min or neighborhood to set many correlations to 0, as not all specifications will result in a positive definite matrix. In particular, sharp drop-offs tend to result in non-positive definite matrices.

Examples

```
## examples
correlation_builder(corr.structure = "ar1", im.res = c(3, 3), rho = 0.5,
                    neighborhood = "round", r = 6, print.all = TRUE)
correlation_builder(corr.structure = "exponential", im.res = c(3, 3),
                    phi = 0.5,
                    neighborhood = "round", r = 3, print.all = TRUE)
correlation_builder(corr.structure = "CS", im.res = c(3, 3),
                    rho = 0.5, print.all = TRUE)
## no "true" zeros, but gets close
c.nr <- correlation_builder(corr.structure = "ar1", neighborhood = "none",</pre>
                    corr.min = NULL, im.res = c(15, 15), rho = 0.5)
length(c.nr[c.nr > 0])
min(c.nr)
## set corr.min gives many zero entries; sparser structure
c.r <- correlation_builder(corr.structure = "ar1", neighborhood = "none",</pre>
                    corr.min = 0.01, im.res = c(15, 15), rho = 0.5)
## raw number > 0
length(c.r[c.r > 0])
## proportion > 0
length(c.r[c.r > 0]) / length(c.nr)
```

corr_fun

Specify the Correlation Function between Two Locations

Description

This is primarily for use within correlation builder, and may be altered/expanded to handle more complicated correlation functions if desired.

Usage

```
corr_fun(
  corr.structure,
  im.res,
```

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```
corr.min = NULL,
rho = NULL,
phi = NULL,
neighborhood = "none",
round.d = FALSE,
w = NULL,
h = NULL,
r = NULL,
i,
j,
k,
v
```

Arguments

 $\verb|corr.structure| One of "ar1", "exponential", "gaussian", or "CS". Correlations between$

locations i and j are rho^d for corr.structure = "ar1", exp(-phi*d) for corr.structure = "exponential", $exp(-phi*d^2)$ for corr.structure = "gaussian",

and rho when corr.structure = "CS". Note that d is the Euclidean distance

between locations i and j.

im.res A vector defining the dimension of spatial data. The first entry is the number of

rows and the second entry is the number of columns.

corr.min Scalar value to specify the minimum non-zero correlation. Any correlations

below corr.min are set to 0. Especially for high image resolution using this option can result in a sparser covariance matrix, which may significantly speed up draws when using spam. This option is preferred to using neighborhood and associated arguments when the primary concern is to avoid very small correlations and improve computation efficiency. Default is NULL, which places no

restrictions on the correlations.

rho This is the maximum possible correlation between locations i and j. For all i,j

rho MUST be between -1 and 1.

phi A scalar value greater than 0 that determines the decay of correlation. This argu-

ment is only utilized when corr.structure %in% c("exponential", "gaussian").

neighborhood Defines the neighborhood within which marginal correlations are non-zero. The

default is "none", which allows marginal correlations to extend indefinitely. neighborhood = "round" defines a circular neighborhood about locations and neighborhood = "rectangle" defines a rectangular neighborhood about loca-

tions.

round.d If round.d = TRUE, then d is rounded to the nearest whole number.

w, h If neighborhood = "rectangle" then w and h are the number of locations to

the left/right and above/below a location i that define its neighborhood. Any locations outside this neighborhood have have zero correlation with location i.

r If neighborhood = "round", then if locations i,j are separated by distance $d \geq$

r, the correlation between them is zero.

 $i,\,j,\,k,\,v$ These are the coordinates for the two locations. Location 1 has coordinates (i,j)

and location 2 has coordinates (k, v).

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Value

A single element vector containing the correlation between spatial locations with indices (i, j) and

Examples

```
## examples
corr_fun(corr.structure = "ar1", im.res = c(3, 3), rho = 0.5,
        neighborhood = "round", r = 6, i = 1, j = 2, k = 2, v = 3)
corr_fun(corr.structure = "ar1", im.res = c(3, 3), rho = 0.5,
        neighborhood = "rectangle", w = 1, h = 1,
        i = 1, j = 2, k = 2, v = 3
```

generate_grid

Convert a 2D Space to Grid Coordinates

Description

Input the limits of a 2D space and the desired image resolution, then the function outputs the appropriate grid/lattice coordinates.

Usage

```
generate_grid(im.res, xlim = c(0, 1), ylim = c(0, 1))
```

Arguments

A vector specifying the dimension/resolution of the image. The first entry is im.res the number of 'rows' in the lattice/image, and the second entry is the number of

columns' in the lattice/image.

These are the 2D image limits. Defaults for both are c(0, 1). It is not recxlim, ylim

ommended to alter these arguments unless changing the limits has a specific

practical utility.

Value

A data frame whose first column is x-coordinates and whose second column is y-coordinates.

```
generate_multinom_probs
```

Generate Probabilities for Multinomial Draws

Description

Obtain probabilities for each category of a multinomial distribution based on covariate and parameter values based on the logit models for the multinomial distribution.

Usage

```
generate_multinom_probs(V = NULL, B = NULL, X = NULL, X.incl.X0 = FALSE)
```

Arguments

V	A numeric value stating the number of categories desired.
В	A list, each element of which contains a parameter vector. The list should have length V – 1, i.e., should contain parameter values associated with all categories except the reference category, following Agresti (2007). Alternatively, B may be a list of length V if one desires to specify parameters for every category, i.e., the over-parameterized model used in Friedman (2010).
Χ	A matrix, each row of which contains subject covariate/predictor values.
X.incl.X0	Logical. When TRUE, X should contain column of 1's for the intercept. Otherwise, a column of 1's is generated internally. Default is FALSE.

Value

A matrix containing subject-specific probabilities for each category of the multinomial distribution. The number of rows equals nrow(X) and the number of columns equals V.

References

Agresti A (2007). An Introduction to Categorical Analysis, 2nd edition. John Wiley & Sons, Hoboken, New Jersey.

Friedman J, Hastie T, Tibshirani R (2010). "Regularization paths for generalized linear models via coordinate descent." *Journal of Statistical Software*, **33**, 1-22. doi:10.18637/jss.v033.i01.

```
## number of categories
vt <- 3

## covariate values
xt <- matrix(rnorm(10 * 2), ncol = 2, nrow = 10)

## list of parameter vectors
bt <- list(b1 = c(1, 0.25, -0.25),</pre>
```

inf_2D_image

inf_2D_image

Display Inference Results for 2D Predictors

Description

Provide graphics for spatial extent of predictor parameters, rejections and/or the truth/falsity of the rejections.

Usage

```
inf_2D_image(
  rejections = NULL,
  B = NULL,
  im.res,
  test.statistic = NULL,
  reject.threshold = NULL,
  binarize.B = TRUE,
  grid.color = "grey",
  n.colors = length(unique(B)),
  B.incl.B0 = TRUE,
  plot.title = TRUE,
  manual.title = NULL,
  title.size = 1
)
```

Arguments

rejections

A binary vector; rejection[i] = 1 means the null hypothesis is rejected for parameter B[i], whereas rejection[i] = 0 means that the null hypothesis was not rejected for parameter B[i].

В

A vector of "true" parameter values. For inference purposes, this can be a vector of actual parameter values, or a binary vector indicating non-zero status.

inf_2D_image

	im.res	A vector defining the dimension of spatial data. The first entry is the number of rows and the second entry is the number of columns.
	test.statistic	A vector of test statistics; e.g., t-statistics or p-values that are used to determine whether or not to reject the null hypothesis.
reject.threshold		
		A list whose first element is the rejection criteria, e.g., the minimum t-statistic or maximum p-value for which to reject the null hypothesis. The second element is one of c("greater", "less", "2-tailed"), which tell the function to reject when the values in test.statistic are greater than or less than the threshold, the test is a 2-tailed, respectively. In the latter case the function internally calculates the upper or lower threshold needed for the 2-tailed test.
	binarize.B	Either TRUE (default) or FALSE. When binarize.B = TRUE the parameter vector is converted to a binary vector where 1 indicates non-zero parameter and 0 indicates zero-valued parameter.
	grid.color	Specify the color for the grid lines.
	n.colors	Determines the number of colors in the printed image. Default is length(unique(B)), but it is recommended to use trial and error to determine the ideal setting for specific situations.
	B.incl.B0	If B.incl.B0 = TRUE then the first entry should be the intercept, B0. B.incl.B0 = FALSE indicates that the first entry of B is not an intercept.
	plot.title	When plot.title = TRUE a title accompanies the output graph, and plot.title = FALSE suppresses the title.
	manual.title	When plot.title = TRUE, use manual.title to specify a title manually.
	title.size	Specifies the size of the title text. This is based on cex.main within the image()

Value

An image depicting the spatial extent of some image characteristic.

function. Default is 1.

Note

If both rejections and B are specified then the function provides an image with separate color each for:

- No rejection and B[i] = 0 (i.e. True Negative).
- No rejection and B[i] != 0 (i.e. False Negative).
- Rejection and B[i] = 0 (i.e. False Positive).
- Rejection and B[i] != 0 (i.e. True Positive).

inf_2D_image

```
B0 = 0, B. values = 1:9,
                    output.indices = FALSE)
## co-opt beta builder to get rejections
rejex <- beta_builder(row.index = c(rep(4, 3), rep(5, 3), rep(6, 3)),
                      col.index = rep(c(4, 5, 6), 3),
                      im.res = c(10, 10), index.type = "manual",
                      B0 = 0, B.values = rep(1, 9),
                      output.indices = FALSE)[-1]
rejex.sm2 <- beta_builder(row.index = 5:6, col.index = 5:6,</pre>
                          im.res = c(10, 10),
                          B0 = 0, B.values = 1,
                          output.indices = FALSE)[-1]
## just B
inf_2D_image(B = Bex, im.res = c(10, 10))
## just rejections
inf_2D_image(rejections = rejex, im.res = c(10, 10))
## both B and rejections
inf_2D_image(rejections = rejex, B = Bex, im.res = c(10, 10))
inf_2D_image(rejections = rejex.sm2, B = Bex, im.res = c(10, 10))
## larger dimension example
Bex2 <- beta_builder(row.index = 5:15, col.index = 16:20,</pre>
                     im.res = c(50, 50), B0 = 0,
                     B.values = 1:(length(5:15) * length(16:20)),
                     index.type = "rectangle",
                     output.indices = FALSE)
rejex2 <- beta_builder(row.index = 13:21, col.index = 30:41,</pre>
                       im.res = c(50, 50), B0 = 0,
                       B.values = rep(1, (length(13:21) * length(30:41))),
                       index.type = "rectangle",
                       output.indices = FALSE)[-1]
rejex3 <- beta_builder(row.index = 5:20, col.index = 16:30,</pre>
                       im.res = c(50, 50), B0 = 0,
                       B.values = rep(1, (length(5:20) * length(16:30))),
                       index.type = "rectangle",
                       output.indices = FALSE)[-1]
rejex4 <- beta_builder(row.index = 5:10, col.index = 16:17,</pre>
                       im.res = c(50, 50), B0 = 0,
                       B.values = rep(1, (length(5:10) * length(16:17))),
                       index.type = "rectangle",
                       output.indices = FALSE)[-1]
## images
inf_2D_image(B = Bex2, im.res = c(50, 50))
inf_2D_image(B = Bex2, im.res = c(50, 50), binarize.B = FALSE)
inf_2D_image(rejections = rejex2, im.res = c(50, 50))
## No TP
inf_2D_image(rejections = rejex2, B = Bex2, im.res = c(50, 50))
## ALL TP
```

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```
inf_2D_image(rejections = Bex2[-1], B = Bex2, im.res = c(50, 50))
## No FN
inf_2D_image(rejections = rejex3, B = Bex2, im.res = c(50, 50))
## No FP, but FN
inf_2D_image(rejections = rejex4, im.res = c(50, 50))
inf_2D_image(B = Bex2, im.res = c(50, 50))
inf_2D_image(rejections = rejex4, B = Bex2, im.res = c(50, 50))
```

make_rejection

Determine rejections

Description

Determine rejections

Usage

```
make_rejection(B, reject.threshold, test.statistic)
```

Arguments

В

A vector of "true" parameter values. For inference purposes, this can be a vector of actual parameter values, or a binary vector indicating non-zero status.

reject.threshold

A list whose first element is the rejection criteria, e.g., the minimum t-statistic or maximum p-value for which to reject the null hypothesis. The second element is one of c("greater", "less", "2-tailed"), which tell the function to reject when the values in test.statistic are greater than or less than the threshold, the test is a 2-tailed, respectively. In the latter case the function internally calculates the upper or lower threshold needed for the 2-tailed test.

test.statistic A vector of test statistics; e.g., t-statistics or p-values that are used to determine whether or not to reject the null hypothesis.

Value

A vector of hypothesis testing rejection indicators, where 1 indicates a rejection and 0 otherwise.

20 neighbors_by_dist

Description

Determine and store neighbors by Euclidean Distance Constraints

Usage

```
neighbors_by_dist(x, y, coords, im.res, r, print.ring = FALSE)
```

Arguments

x, y	are the row and column coordinates, respectively.
coords	A dataframe containing indices and coordinates for the image.
im.res	A vector containing the number of rows and columns, respectively.
r	A scalar value determining the radius within which other locations are neighbors to the current location (x, y) .
print.ring	When print.ring = TRUE, each iteration is shown, with corresponding information regarding the number of neighbors present in each ring. This argument primarily exists to allow the user to test whether the neighborhood structure specified is as desired.

Value

A tibble whose first column contains x indices, second column contains y indices, and third column denotes the current ring about a location.

Note

This function avoids testing all points for being with a certain distance in order to determine neighbor status of a given point by progressively widening a box around the point. Each iteration widens the box by an extra ring, and we only test points in the new ring. If at the end of testing a ring there are no new neighbors then we stop expanding the box and return the neighbors' coordinates. For computational efficiency, this function assumes that all arguments except the current point's coordinates have been specified.

```
## Necessary pre-specified arguments required for the function to work.
## image resoluation + number of spatial predictors
im.res <- c(5, 5)
J <- prod(im.res)

## create predictor indices w/ coordinates
row.id <-rep(1, im.res[2])</pre>
```

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precision_builder

Construct a Precision Matrix

Description

This function constructs the precision matrix for a Conditional Autoregression (CAR).

Usage

```
precision_builder(
   im.res,
   tau = 1,
   alpha = 0.75,
   neighborhood = "ar1",
   weight = "binary",
   phi = 1,
   r = NULL,
   w = NULL,
   digits.Q = 10
)
```

Arguments

im.res

A vector defining the dimension of spatial data. The first entry is the number of rows and the second entry is the number of columns.

tau

A vector containing precision parameters. If of length 1, then all precisions are assumed equal. Otherwise the length of tau should equal the number of variables.

alpha

A scalar value between 0 and 1 that defines the strength of correlations. Note that when alpha = 0 the data are independent and when alpha = 1, the joint distribution is the improper Intrinsic Autoregression (IAR), which cannot be used to generate data. Note also that while alpha does control dependence it is not interpretable as a correlation.

neighborhood

Defines the neighborhood within which conditional correlations are non-zero. This differs from use in correlation_builder, where the neighborhood defines non-zero marginal correlations. The default is "ar1", which creates a

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	neighborhood where the spatial locations directly above, below, left, and right of a location are included in the neighborhood. More complicated neighborhoods can be specified by neighborhood = "round", which defines a circular neighborhood about each location, and neighborhood = "rectangle", which defines a rectangular neighborhood about each location.
weight	Determines how weights are assigned. "distance" assigns weights as the inverse of Euclidean distance times a constant, phi. "binary" assigns weights to 1 for neighbors and 0 otherwise.
phi	When weight = "distance" a constant by which to multiply the inverse of Euclidean distance. Defaults to 1, and must exceed 0.
r	If neighborhood = "round", then locations i,j are separated by distance $d \geq r$ are conditionally independent.
w, h	If neighborhood = "rectangle" then w and h are the number of locations to the left/right and above/below a location i that define its neighborhood. Any locations outside this neighborhood are conditionally independent of the specified location.
digits.Q	Determines the number of digits to round entries in the precision matrix. Default is 10.

Details

This formulation of the CAR model is based on a formulation found in (Banerjee et al. 2015) where the joint distribution of the of the conditionally specified random variables are assumed to be $N(0, [diag(tau^2)(D-alphaW)]^{-1})$ and all neighbors are weighted 1. When weights other than 1 are desired, the joint distribution is $N(0, [diag(tau^2)D(I-alphaD^{-1}W)]^{-1})$, e.g. as in (Jin et al. 2005).

Value

A (precision) matrix.

References

Banerjee S, Carlin BP, Gelfand AE (2015). *Hierarchical Modeling and Analysis for Spatial Data*, Second edition. Chapman & Hall/CRC, Boca Raton, Florida.

Jin X, Carlin BP, Banerjee S (2005). "Generalized Hierarchical Multivariate CAR Models for Areal Data." *Biometrics*, **61**(4), 950-961. doi:10.1111/j.15410420.2005.00359.x.

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proximity_builder

Generate a Proximity Matrix

Description

Generates a proximity matrix where non-zero entries are the weights associated with neighbors, and zero entries are not neighbors.

Usage

```
proximity_builder(
   im.res,
   neighborhood = "ar1",
   type = c("sparse", "full"),
   weight = "binary",
   phi = 1,
   r = NULL,
   h = NULL,
   w = NULL,
   include.coords = FALSE,
   print.im = FALSE
)
```

Arguments

im.res A vector defining the dimension of spatial data. The first entry is the number of

rows and the second entry is the number of columns.

neighborhood Determines how to assign neighbor status to locations; i.e. 1 for neighbors,

0 otherwise. type = "round" assigns neighbor status to locations within radius r. type = "ar1" assigns 1 to locations directly above or beside. type = "rectangle" assigns neighbor status to locations within w units to the left or

right and h units up or down.

type Specifies either sparse (type = "sparse") or full (type = "full") proximity

matrix.

weight Determines how weights are assigned. "distance" assigns weights as the in-

verse of Euclidean distance times a constant, phi. "binary" assigns weights to

1 for neighbors and 0 otherwise.

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when weight = "distance" a constant by which to multiply the inverse of Euclidean distance. Defaults to 1.
 when neighborhood = "round", r specifies the radius within which other locations are neighbors. When neighborhood = "rectangle", w and h specify the number of units to the left/right and above/below the location are to be counted as neighbors.
 include.coords If type = "sparse" and include.coords = TRUE, then the coordinates of neighbors are returned along with their indices.
 print.im Allows user to print the 2D "image" matrix with index labels to visually verify that the proximity matrix is as expected.

Value

A (proximity) matrix.

```
## Not run:
## adjacency matrix with sparse structure (i.e., 2 columns)
## and ar1 neighborhood
sp.ar1 <- proximity_builder(im.res = c(3, 3),</pre>
                             weight = "binary",
                             neighborhood = "ar1",
                             type = "sparse")
## adjacency matrix with full structure
## (i.e., prod(im.dim) rows & columns) and ar1 neighborhood
full.ar1 <- proximity_builder(im.res = c(3, 3),</pre>
                               weight = "binary",
                               neighborhood = "ar1",
                               type = "full")
## proximity matrix weighted by distance (sparse)
sp.rnd <- proximity_builder(im.res = c(3, 3),</pre>
                             weight = "distance",
                             neighborhood = "round", r = 2,
                             type = "sparse",
                             include.coords = TRUE)
## proximity matrix weighted by distance (full)
full.rnd <- proximity_builder(im.res = c(3, 3),</pre>
                               weight = "distance",
                               neighborhood = "round", r = 2,
                               type = "full")
## End(Not run)
```

25 sample_FP_Power

sample_FP_Power

Obtain Sample False Positive Rates and Power

Description

This function calculates sample FDR, FWER, and Power for large numbers of predictors, given a vector of "true" parameter values and a vector of associated rejections. In the case that more than 1 predictor has a "true" non-zero parameter, then Power is defined as the proportion/percentage of those "true" parameters identified.

Usage

```
sample_FP_Power(
  rejections = NULL,
  FP = NULL,
  TP = NULL
  test.statistic = NULL,
  reject.threshold = NULL,
 B = NULL
 B.incl.B0 = TRUE,
  full.summary = FALSE
)
```

Arguments

rejections

A binary vector; rejection[i] = 1 means the null hypothesis is rejected for parameter B[i], whereas rejection[i] = 0 means that the null hypothesis was not rejected for parameter B[i].

FP, TP

Binary vectors of false positive and true positive indicators, respectively. FP[i] = 1 means the null hypothesis was incorrectly rejected, and TP[i] = 1 means the null hypothesis was correctly rejected. If either argument is NULL, then these vectors are computed; this is the default setting.

test.statistic A vector of test statistics; e.g., t-statistics or p-values that are used to determine whether or not to reject the null hypothesis.

reject.threshold

A list whose first element is the rejection criteria, e.g., the minimum t-statistic or maximum p-value for which to reject the null hypothesis. The second element is one of c("greater", "less", "2-tailed"), which tell the function to reject when the values in test.statistic are greater than or less than the threshold, the test is a 2-tailed, respectively. In the latter case the function internally calculates the upper or lower threshold needed for the 2-tailed test.

В

A vector of "true" parameter values. For inference purposes, this can be a vector of actual parameter values, or a binary vector indicating non-zero status.

B.incl.B0

If B. incl. B0 = TRUE then the first entry should be the intercept, B0. B. incl. B0 = FALSE indicates that the first entry of B is not an intercept.

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full.summary

If full.summary = TRUE then the total numbers of rejections, false positives, true positives, and non-zero parameters are output along with FDR, FWER, and Power; otherwise, only FDR, FWER, and Power are output.

Value

A data frame with columns for sample FDR, FWER, and Power.

Note

The default operating approach is that the null hypothesis is B[i] = 0 for each parameter. If other hypotheses are being tested then B should be converted to a binary vector indicating whether the null hypothesis *should* have been rejected.

```
## example 1
## rejection vector
rej.ex <- c(0, 1, 1, 0, 0, 1, 0)
## false positive vector
fp.ex <-c(0, 0, 1, 0, 0, 0, 0)
## true positive vector
tp.ex <-c(0, 1, 0, 0, 0, 1, 0)
## parameter vector
par.ex <- c(0, 4, 0, 0, 3, 9, 0)
sample_FP_Power(rej.ex, fp.ex, tp.ex, par.ex, B.incl.B0 = FALSE)
## Function can calculate TP and FP vectors
sample_FP_Power(rejections = rej.ex,
               FP = NULL, TP = NULL,
               B = par.ex, B.incl.B0 = FALSE)
## example 2: sum(FP, TP) must equal sum(rejections) or
## function stops execution
rej.ex2 <- c(0, 1, 0, 0, 0, 1, 0)
fp.ex2 <- c(0, 0, 1, 0, 0, 0, 0)
tp.ex2 < -c(0, 1, 0, 0, 0, 1, 0)
par.ex2 <- c(0, 4, 0, 0, 3, 9, 0)
## Not run: sample_FP_Power(rej.ex2,
                         fp.ex2, tp.ex2, par.ex2,
                         B.incl.B0 = FALSE)
## End(Not run)
## example 3: calculate rejections from vector of test statistics
zstat < c(-0.5, 1.98, 2.01, 1.45, -1.99)
# 2-tailed
sample_FP_Power(test.statistic = zstat,
                reject.threshold = list(1.96, "2-tailed"),
```

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```
 B = c(0, 0, 4, 1, -2), \ B.incl.B0 = FALSE) \\ \# \ 1-tailed (upper) \\ sample_FP_Power(test.statistic = zstat, \\ reject.threshold = list(1.96, "greater"), \\ B = c(0, 0, 4, 1, -2), \ B.incl.B0 = FALSE) \\ \#\# \ p-value \\ sample_FP_Power(test.statistic = c(0.44, 0.04, 0.01, 0.06, 0.02), \\ reject.threshold = list(0.05, "less"), \\ B = c(0, 0, 4, 1, -2), \ B.incl.B0 = FALSE) \\
```

sim2D_binarymap

Generate a Binary Map via the Boolean Method

Description

Use a Homogenous Poisson Process to generate random "events", a uniform distribution to generate circles of random radii about the events, and take the union to obtain a random set. This is mapped onto a lattice to obtain a binary map.

Usage

```
sim2D_binarymap(
 Ν,
 xlim = c(0, 1),
 ylim = c(0, 1),
  im.res,
  radius.bounds = c(0.02, 0.1),
 lambda = 50,
  random.lambda = FALSE,
  lambda.sd = 10,
  lambda.bound = NULL,
 prior = "gamma",
  sub.area = FALSE,
 min.sa = c(0.1, 0.1),
 \max.sa = c(0.3, 0.3),
 radius.bounds.min.sa = c(0.02, 0.05),
  radius.bounds.max.sa = c(0.08, 0.15),
  print.subj.sa = FALSE,
 print.lambda = FALSE,
 print.iter = FALSE,
  store.type = "list",
  output.randset = FALSE
)
```

Arguments

Ν

A scalar value determining the number of images to create.

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xlim, ylim These are the 2D image limits. Defaults for both are c(0, 1). It is not rec-

ommended to alter these arguments unless changing the limits has a specific

practical utility.

im.res A vector specifying the dimension/resolution of the image. The first entry is

the number of 'rows' in the lattice/image, and the second entry is the number of

columns' in the lattice/image.

radius.bounds A 2-element vector whose first and second entries determine the minimum and

maximum radius sizes, respectively; these will be the bounds of the uniform distribution used to draw the radii. If sub.area = TRUE, then use radius.bounds.min.sa

and radius.bounds.max.sa.

lambda A scalar value specifying the mean/intensity value of the Poisson process. If

random.lambda = FALSE then this is the parameter used to generate the binary image for each subject. If random.lambda = TRUE, then this is the mean param-

eter in the distribution used to draw subject-specific lambda.

random.lambda random.lambda = TRUE allows the lambda (mean/intensity) parameter in the

Poisson process to vary randomly by subject.

lambda.sd Only utilized when random.lambda = TRUE, and specifies the standard deviation

in the distribution used to draw subject-specific lambda.

lambda.bound Only utilized when random.lambda = TRUE, and allows the user to specify a

lower and upper bound for the subject-specific lambda; if the randomly selected value is outside of this range, then another draw is taken. This continues until a value is selected within the specified bounds. If no bounds are desired then

specify lambda.bound = NULL.

prior Only utilized when random. lambda = TRUE, and specifies the distribution from

which to draw the subject-specific lambda. Options are c("gaussian", "gamma").

sub.area When sub.area = TRUE, a random sub-section of the image is chosen, within

which the Poisson process is used to generate the binary image.

min.sa, max.sa Only utilized when sub.area = TRUE, and determines the width and height of the minimum and maximum sub-areas; e.g., if min.sa = c(0.1, 0.1), then the

smallest possible random sub-area is a 0.1 x 0.1 square.

radius.bounds.min.sa, radius.bounds.max.sa

Only utilized when $\verb+sub+$ area = TRUE, and specifies radius bounds for the minimum and maximum $\verb+sub+$ areas, respectively. This information is used to adaptoral to the sub-triangle of the sub-triangle

tively alter the bounds in between the minimum and maximum sub-areas.

print.subj.sa, print.lambda, print.iter

These arguments are either TRUE or FALSE, and define print options for checking that the function is working as the user intends. print.subj.sa = TRUE prints the x-and y-limits for each subject's sub-area. print.lambda = TRUE prints each subject's mean and realized events; the means will be the same unless random.lambda = TRUE, but the number of realized events will always vary.

print.iter = TRUE is only used when random.lambda = TRUE and is.null(lambda.bound)

= FALSE, and shows iterations for re-drawing when the randomly selected inten-

sity is outside the specified bounds.

one of c("list", "matrix", "all"). When store.type = "list", the output is a list where each element is a matrix defining a subject image. If store.type

= "matrix", then the images are vectorized by row and each row of the output matrix contains an image vector for a single subject.

output.randset Logical. When TRUE, stores the data frame of original draws from the HPPP and and random radii from sim2D_RandSet_HPPP(). This data frame is stored in the first element of the output list named randset. The second element of the output list is a list/matrix of the final subject images depending on store.type and named images.

Value

A list; each element is a matrix of zeroes and ones.

References

Cressie N, Wikle CK (2011). Statistics for Spatio-Temporal Data, Wiley Series in Probability and Statistics. John Wiley & Sons, Hoboken, NJ.

Examples

```
bin_ims <- sim2D_binarymap(N = 5, im.res = c(10, 10), store.type = "list",</pre>
                           lambda = 50, sub.area = TRUE,
                           min.sa = c(0.10, 0.10), max.sa = c(0.5, 0.5),
                            radius.bounds.min.sa = c(0.015, 0.04),
                            radius.bounds.max.sa = c(0.041, 0.06))
rotate = function(x){
  t(apply(x, 2, rev))
for (i in 1:length(bin_ims)) {
  image(rotate(bin_ims[[i]]),
        col = c("white", "darkgreen"),
        axes = FALSE)
  box()
  grid(nx = 10, ny = 10, col = "black",
       1ty = 1
}
```

sim2D_RandSet_HPPP

Generate a Random Set Using a Poisson Process and Random Radii About Events

Description

A random set is generated by using a Poisson process in 2D space to choose 'event' locations, about which a circle of random radius is 'drawn'. The union of the circles defines ultimately defines the set.

Usage

```
sim2D_RandSet_HPPP(
 Ν,
  xlim = c(0, 1),
 ylim = c(0, 1),
  radius.bounds = c(0.02, 0.1),
  lambda = 50.
  lambda.sd = 10.
  lambda.bound = NULL,
 prior = "gamma",
  random.lambda = FALSE,
  sub.area = FALSE,
 min.sa = c(0.1, 0.1),
 max.sa = c(0.3, 0.3),
  radius.bounds.min.sa = c(0.02, 0.05),
  radius.bounds.max.sa = c(0.08, 0.15),
  print.subj.sa = FALSE,
 print.lambda = FALSE,
  print.iter = FALSE
)
```

Arguments

N A scalar value determining the number of images to create.

xlim, ylim These are the 2D image limits. Defaults for both are c(0, 1). It is not recommended to alter these arguments unless changing the limits has a specific

practical utility.

radius.bounds A 2-element vector whose first and second entries determine the minimum and

maximum radius sizes, respectively; these will be the bounds of the uniform distribution used to draw the radii. If sub.area = TRUE, then use radius.bounds.min.sa

and radius.bounds.max.sa.

lambda A scalar value specifying the mean/intensity value of the Poisson process. If

random.lambda = FALSE then this is the parameter used to generate the binary image for each subject. If random.lambda = TRUE, then this is the mean parameter used to the little of the lambda.

eter in the distribution used to draw subject-specific lambda.

lambda.sd Only utilized when random.lambda = TRUE, and specifies the standard deviation

in the distribution used to draw subject-specific lambda.

lambda.bound Only utilized when random.lambda = TRUE, and allows the user to specify a

lower and upper bound for the subject-specific lambda; if the randomly selected value is outside of this range, then another draw is taken. This continues until a value is selected within the specified bounds. If no bounds are desired then

specify lambda.bound = NULL.

only utilized when random.lambda = TRUE, and specifies the distribution from

which to draw the subject-specific lambda. Options are c("gaussian", "gamma").

random.lambda random.lambda = TRUE allows the lambda (mean/intensity) parameter in the

Poisson process to vary randomly by subject.

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sub.area

When sub.area = TRUE, a random sub-section of the image is chosen, within which the Poisson process is used to generate the binary image.

min.sa, max.sa

Only utilized when sub.area = TRUE, and determines the width and height of the minimum and maximum sub-areas; e.g., if min.sa = c(0.1, 0.1), then the smallest possible random sub-area is a 0.1×0.1 square.

radius.bounds.min.sa, radius.bounds.max.sa

Only utilized when sub.area = TRUE, and specifies radius.bounds for the minimum and maximum sub-areas, respectively. This information is used to adaptively alter the bounds in between the minimum and maximum sub-areas.

```
print.subj.sa, print.lambda, print.iter
```

These arguments are either TRUE or FALSE, and define print options for checking that the function is working as the user intends. print.subj.sa = TRUE prints the x-and y-limits for each subject's sub-area. print.lambda = TRUE prints each subject's mean and realized events; the means will be the same unless random.lambda = TRUE, but the number of realized events will always vary. print.iter = TRUE is only used when random.lambda = TRUE and is.null(lambda.bound) = FALSE, and shows iterations for re-drawing when the randomly selected intensity is outside the specified bounds.

Value

A dataframe with columns for subject ID, x-coordinates, y-coordinates, and associated radii.

References

Cressie N, Wikle CK (2011). *Statistics for Spatio-Temporal Data*, Wiley Series in Probability and Statistics. John Wiley & Sons, Hoboken, NJ.

 sim_MVN_X

Simulate Spatially Correlated MVN Data

Description

Takes N draws from a Multivariate Normal (MVN) distribution using either base R or the R package spam. This function requires the Cholesky decomposition of the desired covariance matrix.

Usage

```
sim_MVN_X(
    N,
    mu = 0,
    L = NULL,
    R = NULL,
    S = NULL,
    Q = NULL,
    use.spam = FALSE,
    use.MASS = FALSE,
```

 sim_MVN_X

```
X.categorical = FALSE,
X.num.categories = 2,
X.category.type = "percentile",
X.percentiles = NULL,
X.manual.thresh = NULL,
X.cat.names = NULL
)
```

Arguments

N The number of draws to take from MVN; i.e., the number of subjects.

mu One of the following:

- A single scalar value for common mean.
- A vector of length nrow(R) (equivalently nrow(R)) containing means for the MVN.

L, R L and R are lower and upper triangular matrices, respectively, and are the Cholesky factor(s) of the desired covariance matrix for the MVN. Obtain L or R via chol_s2Dp() with settings triangle = "lower" or triangle = "upper", respectively. Specify either L or R, but NOT both.

S, Q A covariance or precision matrix respectively. These are for use with spam, and can be extracted from output of chol_s2Dp after choosing return.cov = TRUE or return.prec = TRUE, respectively.

Logical. If use.spam = TRUE then use tools from the R package spam; otherwise, base R functions are employed. For large dimension MVN with sparse correlation structure, spam is recommended; otherwise, base R may be faster. Defaults to FALSE. Requires either the covariance matrix S or precision matrix, Q, that corresponds to the Cholesky factor.

Logical. When TRUE draws X from MVN using myrnorm from MASS. Note that this requires specification of the covariance matrix, S. Specifying the precision matrix instead may slow down the process for large dimensions. Recommended to use spam to generate draws when specifying a precision matrix, Q.

X.categorical Default is X.categorical = FALSE. If X.categorical = TRUE then thresholds are applied to categorize each predictor/image value.

X.num.categories

use.spam

use.MASS

A scalar value denoting the number of categories in which to divide the data.

X.category.type

Tells R how to categorize the data. Options are X.category.type = c("percentile", "manual"). If X.category.type = "percentile" then the data are divided into percentiles based on X.num.categories; e.g. if X.num.categories = 4 then the values are divided into quartiles, and values in Q1 equal 0, between Q1 and Q2 equal 1, between Q2 and Q3 equal 2, and greater than Q3 equal 3. If X.category.type = "manual" then specify the cutoff points with X.manual.thresh.

X.percentiles A vector of percentiles to be used in thresholding when X.categorical = TRUE and X.category.type = "percentile". The length of this vector should equal the number of categories minus one, and all values should be between zero and one.

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X.manual.thresh

A vector containing the thresholds for categorizing the values; e.g. if X.num.categories = 4 and X.manual.thresh = c(-3, 1, 17), then values less than -3 are set to 0, equal or greater than -3 and less than 1 are set to 1, equal or greater than 1 but less than 17 are set to 2, and equal or greater than 17 are set to 3. Note that length(X.manual.thresh) must always equal X.num.categories - 1.

X.cat.names

A vector of category names. If X.cat.names = NULL then the initial integers assigned are left as the values; the names in X.cat.names are assigned in ascending order.

Value

Matrix of dimension N x (nrow(L)) (or equivalently N x (nrow(R))) where each row is draw from MVN, and each column represents a different "variable"; e.g. location in an image.

Note

This function requires the Cholesky decomposition of the desired covariance matrix for the MVN; this allows for using this function in simulating multiple datasets of N MVN draws while only taking the Cholesky decomposition of the covariance matrix once.

References

Furrer R, Sain SR (2010). "spam: A Sparse Matrix R Package with Emphasis on MCMC Methods for Gaussian Markov Random Fields." *Journal of Statistical Software*, **36**(10), 1-25. https://www.jstatsoft.org/v36/i10/.

Ripley BD (1987). Stochastic Simulation. John Wiley & Sons. doi:10.1002/9780470316726.

Rue H (2001). "Fast Sampling of Gaussian Markov Random Fields." *Journal of the Royal Statistical Society B*, **63**, 325-338. doi:10.1111/14679868.00288.

```
## verify MVN with base R
set.seed(732)
Lex <- chol_s2Dp(corr.structure = "ar1",</pre>
                 im.res = c(3, 3),
                 rho = 0.25,
                 sigma = 1,
                 use.spam = FALSE,
                 corr.min = 0.02,
                 triangle = "lower",
                 return.cov = TRUE)
XbR = sim_MVN_X(N = 1000, mu = 0, L = Lex$L)
apply(XbR, 2, mean)
cov(XbR)
Lex$S
## verify MVN with \code{spam}
set.seed(472)
```

sim_Y_Binary_X

Simulate Scalar Outcomes from Simulated Spatially Dependent Binary Predictors

Description

N spatially dependent binary design vectors are simulated using sim2D_binarymap. These design vectors are used to then simulate scalar outcomes that have one of Gaussian, Binomial, or Poisson distributions.

Usage

```
sim_Y_Binary_X(
 Ν,
 В,
  rand.err = 1,
 dist,
  incl.subjectID = TRUE,
  binomial.method = "traditional",
  count.method = "traditional",
  Y.thresh = NULL,
 print.out = FALSE,
  xlim = c(0, 1),
  ylim = c(0, 1),
  im.res,
  radius.bounds = c(0.02, 0.1),
  lambda = 50,
  random.lambda = FALSE,
  lambda.sd = 10,
```

```
lambda.bound = NULL,
  prior = "gamma",
  sub.area = FALSE,
 min.sa = c(0.1, 0.1),
 max.sa = c(0.3, 0.3),
  radius.bounds.min.sa = c(0.02, 0.05),
  radius.bounds.max.sa = c(0.08, 0.15),
  print.subj.sa = FALSE,
  print.lambda = FALSE,
  print.iter = FALSE
)
```

Arguments

Ν A scalar value determining the number of images to create.

В A vector parameter values; i.e. "betas". Note that length(B) must equal p + 1 = n.row * n.col + 1; e.g. for normal outcomes Y = XB + e with Y a scalar outcome and e the random error.

A scalar for the random error variance when dist = "gaussian". rand.err

dist The distribution of the scalar outcome.

- dist = "gaussian" has Y = XB + e, where e N(0, rand.err).
- dist = "binomial" is drawn from eqnBin(XB, XB(1-XB)) using rbinom() when binary.method = "Traditional". If binary.method = "Gaussian", then simulation is based on a cutoff using binary.cutoff.
- dist = "poisson" is drawn from Poisson(XB) using rpois().

incl.subjectID When incl.subjectID = TRUE a column of subject indices is generated. Y. thresh = NULL (default). If binomial.method = "gaussian manual", then Y.thresh should be any scalar real number; values equal or above this cutoff are assigned 1 and values below are assigned 0. If binomial.method = "gaussian" percentile", then values equal or above this percentile are assigned 1, and other wise 0; in this case values should be between 0 and 1. For example, if Y. thresh = 0.9, then the cutoff is the 90th percentile.

binomial.method

One of c("traditional", "gaussian manual", "gaussian percentile"). Only specified when dist = "binomial", and determines whether draws are directly taken from a binomial distribution or if draws are taken from a Multivariate Normal Distribution (in the manner of dist = "gaussian") and thresholds imposed to binarize the outcomes. binomial.method = "gaussian manual" allows the user to specify specific values for categorizing outcomes. binomial.method = "gaussian percentile" allows the user to specify percentiles for binarizing the data. Both approaches use Y. thresh to specify the cutoff value(s). If binomial.method = "gaussian percentile" and Y.thresh = NULL then the median is used as the threshold. If binomial.method = "gaussian manual" and Y. thresh = NULL, then 0 is used as the threshold. Default is binomial.method = "traditional".

count.method

One of c("traditional", "rounding"). When count.method = "traditional", the outcomes are drawn sequentially using rpois(). When count.method =

"traditional", the outcomes are drawn from an MVN, then values less than or equal to 0 are set to 0, and all other values are rounded to the nearest whole

Y.thresh When binomial.method = "traditional"

print.out If print.out = TRUE then print the following for each subject, indexed y:

• X[y] %*% B

• p[y], lambda[y] for Binomial, Poisson, respectively.

This is useful to see the effect of image parameter selection and beta parameter selection on distributional parameters for the outcome of interest.

xlim, ylim These are the 2D image limits. Defaults for both are c(0, 1). It is not recommended to alter these arguments unless changing the limits has a specific

practical utility.

im.res A vector specifying the dimension/resolution of the image. The first entry is

the number of 'rows' in the lattice/image, and the second entry is the number of

columns' in the lattice/image.

radius.bounds A 2-element vector whose first and second entries determine the minimum and

maximum radius sizes, respectively; these will be the bounds of the uniform distribution used to draw the radii. If sub.area = TRUE, then use radius.bounds.min.sa

and radius.bounds.max.sa.

lambda A scalar value specifying the mean/intensity value of the Poisson process. If

random.lambda = FALSE then this is the parameter used to generate the binary image for each subject. If random.lambda = TRUE, then this is the mean param-

eter in the distribution used to draw subject-specific lambda.

random.lambda random.lambda = TRUE allows the lambda (mean/intensity) parameter in the

Poisson process to vary randomly by subject.

lambda.sd Only utilized when random.lambda = TRUE, and specifies the standard deviation

in the distribution used to draw subject-specific lambda.

lambda.bound Only utilized when random.lambda = TRUE, and allows the user to specify a

lower and upper bound for the subject-specific lambda; if the randomly selected value is outside of this range, then another draw is taken. This continues until a value is selected within the specified bounds. If no bounds are desired then

specify lambda.bound = NULL.

prior Only utilized when random.lambda = TRUE, and specifies the distribution from

which to draw the subject-specific lambda. Options are c("gaussian", "gamma").

sub.area When sub.area = TRUE, a random sub-section of the image is chosen, within

which the Poisson process is used to generate the binary image.

min.sa, max.sa Only utilized when sub.area = TRUE, and determines the width and height of

the minimum and maximum sub-areas; e.g., if min. sa = c(0.1, 0.1), then the

smallest possible random sub-area is a 0.1 x 0.1 square.

radius.bounds.min.sa, radius.bounds.max.sa

Only utilized when sub.area = TRUE, and specifies radius.bounds for the minimum and maximum sub-areas, respectively. This information is used to adaptively alter the bounds in between the minimum and maximum sub-areas.

```
print.subj.sa, print.lambda, print.iter
```

These arguments are either TRUE or FALSE, and define print options for checking that the function is working as the user intends. print.subj.sa = TRUE prints the x-and y-limits for each subject's sub-area. print.lambda = TRUE prints each subject's mean and realized events; the means will be the same unless random.lambda = TRUE, but the number of realized events will always vary. print.iter = TRUE is only used when random.lambda = TRUE and is.null(lambda.bound) = FALSE, and shows iterations for re-drawing when the randomly selected intensity is outside the specified bounds.

Value

A data frame where each row consists of a single subject's data. Col 1 is the outcome, Y, and each successive column contains the subject predictor values.

Note

Careful parameter selection, i.e. B, is necessary to ensure that simulated outcomes are reasonable; in particular, counts arising from the Poisson distribution can be unnaturally large.

References

Cressie N, Wikle CK (2011). *Statistics for Spatio-Temporal Data*, Wiley Series in Probability and Statistics. John Wiley & Sons, Hoboken, NJ.

Ripley BD (1987). Stochastic Simulation. John Wiley & Sons. doi:10.1002/9780470316726.

```
## Define non-zero beta values
Bex <- beta_builder(row.index = c(3, 3, 4),</pre>
                     col.index = c(3, 4, 3),
                     im.res = c(5, 5),
                     B0 = 0, B. values = rep(1/3, 3),
                     output.indices = FALSE)
## Simulate Datasets
## parameter values
Nex = 10
set.seed(28743)
Gauss.ex <- sim_Y_Binary_X(N = Nex,</pre>
                            B = Bex,
                            dist = "gaussian",
                            im.res = c(5, 5)
hist(Gauss.ex$Y)
## direct draws from binomial
Bin.ex <- sim_Y_Binary_X(N = Nex,</pre>
                          B = Bex,
                          im.res = c(5, 5),
                          dist = "binomial",
                           print.out = TRUE)
```

 $sim_Y_MVN_X$

```
table(Bin.ex$Y)
```

sim_Y_MVN_X

Simulate Scalar Outcomes from Simulated Spatially Correlated Predictors

Description

N spatially correlated design vectors are simulated from an MVN. These design vectors are used to then simulate scalar outcomes that have one of Gaussian, Binomial, Multinomial or Poisson distributions.

Usage

```
sim_Y_MVN_X(
 Ν,
 В,
 L = NULL
 R = NULL
  S = NULL
  Q = NULL,
  use.spam = TRUE,
 mu = 0,
  rand.err = 1,
  dist = "gaussian",
  V = NULL
  incl.subjectID = TRUE,
  threshold.method = "none",
  Y.thresh = NULL,
 X.categorical = FALSE,
 X.num.categories = 2,
 X.category.type = "percentile",
 X.manual.thresh = NULL,
 X.cat.names = NULL,
  print.out = FALSE
)
```

Arguments

N

The number of draws to take from MVN; i.e., the number of subjects.

В

A vector parameter values; i.e. "betas". Note that length(B) must equal p+1=n.row*n.col+1; e.g. for normal outcomes Y=XB+e with Y a scalar outcome and e the random error. Note that when dist="multinomial" then B should be a list with length equal to V - 1, i.e., should contain parameter values associated with all categories except the reference category. Alternatively, when dist="multinomial" B may be a list of length V if one desires to specify parameters for every category, i.e., the over-parameterized model used in Friedman (2010).

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L, R L and R are lower and upper triangular matrices, respectively, and are the Cholesky factor(s) of the desired covariance matrix for the MVN. Obtain L or R via chol. s2

factor(s) of the desired covariance matrix for the MVN. Obtain L or R via chol_s2Dp() with settings triangle = "lower" or triangle = "upper", respectively. Spec-

ify either L or R, but NOT both.

S, Q A covariance or precision matrix respectively. These are for use with spam, and can be extracted from output of chol_s2Dp after choosing return.cov = TRUE

or return.prec = TRUE, respectively.

use.spam Logical. If use.spam = TRUE then use tools from the R package spam; otherwise, base R functions are employed. For large dimension MVN with sparse correlation structure, spam is recommended; otherwise, base R may be faster. Defaults to FALSE. Requires either the covariance matrix S or precision matrix,

Q, that corresponds to the Cholesky factor.

One of the following:

• A single scalar value for common mean.

 A vector of length nrow(R) (equivalently nrow(R)) containing means for the MVN.

A vector for the random error standard deviation when dist = "gaussian", or thresholding is used to obtain non-Normal draws. Must have length 1 or length N

The distribution of the scalar outcome.

• dist = "gaussian" has Y = XB + e, where e N(0, rand.err).

- dist = "binomial": Y is drawn from a binomial distribution with probability of "success" equal to 1/(1+1/exp(XB)) using rbinom() when binary.method = "traditional". If binary.method = "gaussian", then simulation is based on a cutoff using binary.cutoff.
- dist = "multinomial": Y is drawn from sample() using probabilities generated based on Chapter 6.1.3 of Agresti (2007) when length(B) = V 1 or Friedman (2010) when the length(B) = V. Threshold-based approaches are not currently supported.
- dist = "poisson": Y is drawn from Poisson(exp(XB)) using rpois().

A numeric value stating the number of categories desired when dist = "multinomial".

incl.subjectID When incl.subjectID = TRUE a column of subject indices is generated.
threshold.method

One of "none", "manual", "percentile", "round". When "none" draws from Binomial or Poisson distributions are taken subject-wise using base R functions. For the remaining options, draws are first taken from a Normal distribution and then thresholded. "manual" uses Y. thresh to manually select a cutoff, "percentile" uses Y. thresh to select percentiles used to bin outcomes, and "round" sets values equal or less than 0 to 0, and rounds all positive values to the nearest whole number.

A manual value used to threshold when threshold.method = "manual"; values equal or greater than the cutoff are assigned 1 and all others 0. When threshold.method = "percentile", a percentile to use to bin outcomes.

X.categorical Default is X.categorical = FALSE. If X.categorical = TRUE then thresholds are applied to categorize each predictor/image value.

mu

rand.err

dist

.,

Y.thresh

 $sim_{\underline{Y}}MVN_{\underline{X}}$

X.num.categories

A scalar value denoting the number of categories in which to divide the data.

X.category.type

Tells R how to categorize the data. Options are X.category.type = c("percentile", "manual"). If X.category.type = "percentile" then the data are divided into percentiles based on X.num.categories; e.g. if X.num.categories = 4 then the values are divided into quartiles, and values in Q1 equal 0, between Q1 and Q2 equal 1, between Q2 and Q3 equal 2, and greater than Q3 equal 3. If X.category.type = "manual" then specify the cutoff points with X.manual.thresh.

X.manual.thresh

A vector containing the thresholds for categorizing the values; e.g. if X.num.categories = 4 and X.manual.thresh = c(-3, 1, 17), then values less than -3 are set to 0, equal or greater than -3 and less than 1 are set to 1, equal or greater than 1 but less than 17 are set to 2, and equal or greater than 17 are set to 3. Note that length(X.manual.thresh) must always equal X.num.categories - 1.

X.cat.names

A vector of category names. If X.cat.names = NULL then the initial integers assigned are left as the values; the names in X.cat.names are assigned in ascending order.

print.out

If print.out = TRUE then print the following for each subject, indexed y:

- X[y] %*% B
- p[y], lambda[y] for Binomial, Poisson, respectively.

This is useful to see the effect of image parameter selection and beta parameter selection on distributional parameters for the outcome of interest.

Value

A data frame where each row consists of a single subject's data. Col 1 is the outcome, Y, and each successive column contains the subject predictor values.

Note

Careful parameter selection, i.e. B, is necessary to ensure that simulated outcomes are reasonable; in particular, counts arising from the Poisson distribution can be unnaturally large.

References

Furrer R, Sain SR (2010). "spam: A Sparse Matrix R Package with Emphasis on MCMC Methods for Gaussian Markov Random Fields." *Journal of Statistical Software*, **36**(10), 1-25. https://www.jstatsoft.org/v36/i10/.

Ripley BD (1987). Stochastic Simulation. John Wiley & Sons. doi:10.1002/9780470316726.

Rue H (2001). "Fast Sampling of Gaussian Markov Random Fields." *Journal of the Royal Statistical Society B*, **63**, 325-338. doi:10.1111/14679868.00288.

Agresti A (2007). An Introduction to Categorical Analysis, 2nd edition. John Wiley & Sons, Hoboken, New Jersey.

Friedman J, Hastie T, Tibshirani R (2010). "Regularization paths for generalized linear models via coordinate descent." *Journal of Statistical Software*, **33**, 1-22. doi:10.18637/jss.v033.i01.

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```
## generate precision matrix and take Cholesky decomposition
Rpre <- chol_s2Dp(im.res = c(3, 3), matrix.type = "prec",</pre>
                  use.spam = TRUE, neighborhood = "ar1",
                  triangle = "upper", return.prec = TRUE)
## Generate correlation matrix & take Cholesky decomposition
Rcov <- chol_s2Dp(corr.structure = "ar1", im.res = c(3, 3),</pre>
                  rho = 0.5,
                  triangle = "upper",
                  use.spam = FALSE, neighborhood = "none")
## Define non-zero beta values
Bex <- beta_builder(row.index = c(2, 3),</pre>
                    col.index = c(3, 3),
                    im.res = c(3, 3),
                    B0 = 0, B.values = rep(1, 2),
                    output.indices = FALSE)
## Simulate Datasets
## parameter values
Nex = 100
set.seed(28743)
## with precision matrix
Gauss.exp \leftarrow sim_Y_MVN_X(N = Nex, B = Bex,
                          R = Rpre\$R, Q = Rpre\$Q,
                          dist = "gaussian")
hist(Gauss.exp$Y)
## with covariance matrix
Gauss.exc < sim_Y_MVN_X(N = Nex, B = Bex,
                          R = Rcov\$R, S = Rcov\$S,
                          dist = "gaussian")
hist(Gauss.exc$Y)
## direct draws from binomial
Bin.ex <- sim_Y_MVN_X(N = Nex, B = Bex, R = Rcov$R, S = Rcov$S,
                      dist = "binomial", print.out = TRUE)
table(Bin.ex$Y)
## manual cutoff
Bin.ex2 <- sim_Y_MVN_X(N = Nex, B = Bex,
                        R = Rcov\$R, S = Rcov\$S,
                        dist = "binomial",
                        threshold.method = "manual",
                        Y.thresh = 1.25)
table(Bin.ex2$Y)
## percentile cutoff
Bin.ex3 < - sim_Y_MVN_X(N = Nex, B = Bex,
                        R = Rcov\$R, S = Rcov\$S,
                        dist = "binomial",
                        threshold.method = "percentile",
```

42 within_area

within_area

Determine Whether Lattice Points are Within or Without a Random Set

Description

Determine whether locations in the image/lattice (from generate.grid) are within or without the union of a random set generated by sim2D_HPPP_coords(). If the Euclidean distance between a lattice location and any 'event' is less than the radius about the 'event', then the location is said to be within the random set. Otherwise, it is without the random set.

Usage

```
within_area(grid.centers, radii, event.xcoord, event.ycoord)
```

Arguments

grid.centers Output from generate.grid() that specifies the coordinates of the lattice loca-

tions in native space.

radii A vector of radii values.

event.xcoord, event.ycoord

Paired vectors specifying the x- and y- coordinates, respectively, of each 'event' from the Poisson process.

Value

A data frame with lattice x- and y- coordinates, and a binary vector where 1 indicates the location is within the random set, and 0 indicates the location is without the random set.

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