

# Package ‘spmoran’

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**Type** Package

**Title** Fast Spatial and Spatio-Temporal Regression using Moran Eigenvectors

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**Description** A collection of functions for estimating spatial and spatio-temporal regression models. Moran eigenvectors are used as spatial basis functions to efficiently approximate spatially dependent Gaussian processes (i.e., random effects eigenvector spatial filtering; see Murakami and Griffith 2015 <[doi:10.1007/s10109-015-0213-7](https://doi.org/10.1007/s10109-015-0213-7)>). The implemented models include linear regression with residual spatial dependence, spatially/spatio-temporally varying coefficient models (Murakami et al., 2017, 2024; <[doi:10.1016/j.spasta.2016.12.001](https://doi.org/10.1016/j.spasta.2016.12.001)>, <[doi:10.48550/arXiv.2410.07229](https://doi.org/10.48550/arXiv.2410.07229)>), spatially filtered unconditional quantile regression (Murakami and Seya, 2019 <[doi:10.1002/env.2556](https://doi.org/10.1002/env.2556)>), Gaussian and non-Gaussian spatial mixed models through compositionally-warping (Murakami et al. 2021, <[doi:10.1016/j.spasta.2021.100520](https://doi.org/10.1016/j.spasta.2021.100520)>).

**License** GPL (>= 2)

**Encoding** UTF-8

**Imports** sf, fields, vegan, Matrix, doParallel, foreach, ggplot2, spdep, rARPACK, RColorBrewer, splines, FNN, methods

**Suggests** R.rsp, spData (>= 2.3.1)

**VignetteBuilder** R.rsp

**URL** <https://github.com/dmuraka/spmoran>

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addlearn_local	<i>Additional learning of local processes and prediction for large samples</i>
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Description

This function performs an additional learning of local variations in spatially varying coefficients (SVC). While the SVC model implemented in `resf_vc` or `besf_vc` can be less accurate for large samples (e.g.,  $n > 5,000$ ) due to a degeneracy/over-smoothing problem, this additional learning mitigates this problem by synthesizing/averaging the model with local SVC models. The resulting spatial prediction implemented in this function is expected to be more accurate than the `resf_vc` function.

Note that this function is not yet supported for spatio-temporal models with `!is.null(meig$coords_z)`.

Usage

```
addlearn_local( mod, meig0 = NULL, x0 = NULL, xconst0=NULL, xgroup0=NULL,
                cl_num=NULL, cl=NULL, parallel=FALSE, ncores=NULL )
```

**Arguments**

mod	Output from <code>resf</code> , <code>resf_vc</code> or <code>besf_vc</code> function
meig0	Moran eigenvectors at prediction sites. Output from <code>meigen0</code>
x0	Matrix of explanatory variables at prediction sites whose coefficients are allowed to vary across geographical space ( $N_0 \times K$ ). Default is NULL
xconst0	Matrix of explanatory variables at prediction sites whose coefficients are assumed constant (or NVC) across space ( $N_0 \times K_{const}$ ). Default is NULL
xgroup0	Matrix of group indeces at prediction sites that may be group IDs (integers) or group names ( $N_0 \times K_g$ ). Default is NULL
cl_num	Number of local sub-models being aggregated/averaged. If NULL, the number is determined so that the number of samples per sub-model equals approximately 600. Default is NULL
cl	Vector of cluster ID for each sample ( $N \times 1$ ). If specified, the local sub-models are given by this ID. If NULL, k-means clustering based on spatial coordinates is performed to obtain spatial clusters each of which contain approximately 600 samples. Default is NULL
parallel	If TRUE, the model is estimated through parallel computation. The default is FALSE
ncores	Number of cores used for the parallel computation. If ncores = NULL and parallel = TRUE, the number of available cores - 2 is used. Default is NULL

**Value**

b_vc	Matrix of estimated spatially varying coefficients (SVCs) on x ( $N \times K$ )
bse_vc	Matrix of standard errors for the SVCs on x ( $N \times k$ )
z_vc	Matrix of z-values for the SVCs on x ( $N \times K$ )
p_vc	Matrix of p-values for the SVCs on x ( $N \times K$ )
c	Matrix with columns for the estimated coefficients on xconst, their standard errors, z-values, and p-values ( $K_c \times 4$ )
b_g	List of $K_g$ matrices with columns for the estimated group effects, their standard deviations, and t-values
s	List of 2 elements summarizing variance parameters characterizing SVCs of each local sub-model. The first element contains standard deviations of each SVCs while the second element contains their Moran's I values that are scaled to take a value between 0 (no spatial dependence) and 1 (strongest positive spatial dependence). Based on Griffith (2003), the scaled Moran's I value is interpretable as follows: 0.25-0.50:weak; 0.50-0.70:moderate; 0.70-0.90:strong; 0.90-1.00:marked
s_global	The same variance parameters for the global sub-model
s_g	Vector of standard deviations of the group effects
e	Error statistics. It includes residual standard error (resid_SE), adjusted conditional R <sup>2</sup> (adjR <sup>2</sup> (cond)), restricted log-likelihood (rlogLik), Akaike information criterion (AIC), and Bayesian information criterion (BIC)

pred	Matrix of predicted values for y (pred) and their standard errors (pred_se) (N x 2)
resid	Vector of residuals (N x 1)
cl	Vector of cluster ID being used (N x 1)
pred0	Matrix of predicted values for y (pred) and their standard errors (pred_se) at prediction sites (N_0 x 2)
b_vc0	Matrix of estimated spatially varying coefficients (SVCs) at prediction sites (N_0 x K)
bse_vc0	Matrix of standard errors for the SVCs at prediction sites (N_0 x k)
z_vc0	Matrix of z-values for the SVCs at prediction sites (N x K)
p_vc0	Matrix of p-values for the SVCs at prediction sites (N x K)
other	List of other outputs, which are internally used

### Author(s)

Daisuke Murakami

### References

Murakami, D., Sugawara, S., T., Seya, H., and Griffith, D.A. (2024) Sub-model aggregation-based scalable eigenvector spatial filtering: application to spatially varying coefficient modeling. Geographical Analysis, DOI: 10.1111/gean.12393.

### See Also

[resf](#), [resf\\_vc](#), [besf\\_vc](#)

### Examples

```
require(spdep)
data(house)
dat0 <- data.frame(house@coords,house@data)
dat <- dat0[dat0$yrbuilt>=1980,]

##### purpose 1: improve SVC modeling accuracy #####
##### (i.e., addressing the over-smoothing problem) #
y <- log(dat[, "price"])
x <- dat[,c("age", "rooms")]
xconst <- dat[,c("lotsize", "s1994", "s1995", "s1996", "s1997", "s1998")]
coords <- dat[,c("long", "lat")]
meig <- meigen_f( coords )

## Not run
# res0 <- resf_vc(y = y,x = x, xconst = xconst, meig = meig)
# res <- addlearn_local(res0) # It adjusts SVCs to model local patterns
# res

##### parallel version for very large samples (e.g., n >100,000)
# bes0 <- besf_vc(y = y,x = x, xconst = xconst, coords=coords)
```



x	Matrix of explanatory variables (N x K)
nvc	If TRUE, NVCs are assumed on x. Otherwise, constant coefficients are assumed. Default is FALSE
nvc_sel	If TRUE, type of coefficients (NVC or constant) is selected through a BIC (default) or AIC minimization. If FALSE, NVCs are assumed across x. Alternatively, nvc_sel can be given by column number(s) of x. For example, if nvc_sel = 2, the coefficient on the second explanatory variable in x is NVC and the other coefficients are constants. The Default is TRUE
coords	Matrix of spatial point coordinates (N x 2)
s_id	Optional. ID specifying groups modeling spatially dependent process (N x 1). If it is specified, group-level spatial process is estimated. It is useful. e.g., for multilevel modeling (s_id is given by the group ID) and panel data modeling (s_id is given by individual location id). Default is NULL
covmodel	Type of kernel to model spatial dependence. The currently available options are "exp" for the exponential kernel, "gau" for the Gaussian kernel, and "sph" for the spherical kernel
enum	Number of Moran eigenvectors to be used for spatial process modeling (scalar). Default is 200
method	Estimation method. Restricted maximum likelihood method ("reml") and maximum likelihood method ("ml") are available. Default is "reml"
penalty	Penalty to select type of coefficients (NVC or constant) to stabilize the estimates. The current options are "bic" for the Bayesian information criterion-type penalty ( $N \times \log(K)$ ) and "aic" for the Akaike information criterion (2K) (see Muller et al., 2013). Default is "bic"
nvc_num	Number of basis functions used to model NVC. An intercept and nvc_num natural spline basis functions are used to model each NVC. Default is 5
maxiter	Maximum number of iterations. Default is 30
bsize	Block/batch size. bsize x bsize elements are iteratively processed during the parallelized computation. Default is 4000
ncores	Number of cores used for the parallel computation. If ncores = NULL, the number of available cores - 2 is detected and used. Default is NULL

### Value

b	Matrix with columns for the estimated coefficients on x, their standard errors, z-values, and p-values (K x 4). Effective if nvc = FALSE
c_vc	Matrix of estimated NVCs on x (N x K). Effective if nvc = TRUE
cse_vc	Matrix of standard errors for the NVCs on x (N x K). Effective if nvc = TRUE
ct_vc	Matrix of t-values for the NVCs on x (N x K). Effective if nvc = TRUE
cp_vc	Matrix of p-values for the NVCs on x (N x K). Effective if nvc = TRUE
s	Vector of estimated variance parameters (2 x 1). The first and the second elements denote the standard deviation and the Moran's I value of the estimated spatially dependent component, respectively. The Moran's I value is scaled to

	take a value between 0 (no spatial dependence) and 1 (the maximum possible spatial dependence). Based on Griffith (2003), the scaled Moran's I value is interpretable as follows: 0.25-0.50:weak; 0.50-0.70:moderate; 0.70-0.90:strong; 0.90-1.00:marked
e	Vector whose elements are residual standard error (resid_SE), adjusted conditional R <sup>2</sup> (adjR2(cond)), restricted log-likelihood (rlogLik), Akaike information criterion (AIC), and Bayesian information criterion (BIC). When method = "ml", restricted log-likelihood (rlogLik) is replaced with log-likelihood (logLik)
vc	List indicating whether NVC are removed or not during the BIC/AIC minimization. 1 indicates not removed whereas 0 indicates removed
r	Vector of estimated random coefficients on Moran's eigenvectors (L x 1)
sf	Vector of estimated spatial dependent component (N x 1)
pred	Vector of predicted values (N x 1)
resid	Vector of residuals (N x 1)
other	List of other outputs, which are internally used

### Author(s)

Daisuke Murakami

### References

- Griffith, D. A. (2003). Spatial autocorrelation and spatial filtering: gaining understanding through theory and scientific visualization. Springer Science & Business Media.
- Murakami, D. and Griffith, D.A. (2015) Random effects specifications in eigenvector spatial filtering: a simulation study. *Journal of Geographical Systems*, 17 (4), 311-331.
- Murakami, D. and Griffith, D.A. (2019) A memory-free spatial additive mixed modeling for big spatial data. *Japan Journal of Statistics and Data Science*. DOI:10.1007/s42081-019-00063-x.

### See Also

[resf](#)

### Examples

```
require(spdep)
data(boston)
y <- boston.c[, "CMEDV" ]
x <- boston.c[,c("CRIM","ZN","INDUS", "CHAS", "NOX","RM", "AGE",
                 "DIS", "RAD", "TAX", "PTRATIO", "B", "LSTAT")]
xgroup <- boston.c[, "TOWN"]
coords <- boston.c[,c("LON", "LAT")]

##### Regression considering spatially dependent residuals
#res <- besf(y = y, x = x, coords=coords)
#res

##### Regression considering spatially dependent residuals and NVC
```

```
##### (coefficients or NVC is selected)
#res2 <- besf(y = y, x = x, coords=coords, nvc = TRUE)

##### Regression considering spatially dependent residuals and NVC
##### (all the coefficients are NVCs)
#res3 <- besf(y = y, x = x, coords=coords, nvc = TRUE, nvc_sel=FALSE)
```

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besf_vc	<i>Spatially and non-spatially varying coefficient (SNVC) modeling for very large samples</i>
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## Description

Parallel and memory-free implementation of SNVC modeling for very large samples. The model estimates residual spatial dependence, constant coefficients, spatially varying coefficients (SVCs), non-spatially varying coefficients (NVC; coefficients varying with respect to explanatory variable value), and SNVC (= SVC + NVC). Type of coefficients can be selected through BIC/AIC minimization. By default, it estimates a SVC model. SNVCs can be mapped just like SVCs. Unlike SVC models, SNVC model is robust against spurious correlation (multicollinearity), so, stable (see Murakami and Griffith, 2020). This function is not yet supported for spatio-temporal modeling.

Note: The SVC model can be less accurate for large samples due to a degeneracy/over-smoothing problem (see Murakami et al., 2023). The [addlearn\\_local](#) is useful to mitigate this problem (See the coding example below).

## Usage

```
besf_vc( y, x, xconst = NULL, coords, s_id = NULL, x_nvc = FALSE, xconst_nvc = FALSE,
         x_sel = TRUE, x_nvc_sel = TRUE, xconst_nvc_sel = TRUE, nvc_num=5,
         method = "reml", penalty = "bic", maxiter = 30, tol = 1e-30,
         covmodel="exp",enum = 200, bsize = 4000, ncores=NULL )
```

## Arguments

y	Vector of explained variables (N x 1)
x	Matrix of explanatory variables with spatially varying coefficients (SVC) (N x K)
xconst	Matrix of explanatory variables with constant coefficients (N x K_c). Default is NULL
coords	Matrix of spatial point coordinates (N x 2)
s_id	Optional. ID specifying groups modeling spatially dependent process (N x 1). If it is specified, group-level spatial process is estimated. It is useful for multilevel modeling (s_id is given by the group ID) and panel data modeling (s_id is given by individual location id). Default is NULL
x_nvc	If TRUE, SNVCs are assumed on x. Otherwise, SVCs are assumed. Default is FALSE



xconst_nvc	If TRUE, NVCs are assumed on xconst. Otherwise, constant coefficients are assumed. Default is FALSE
x_sel	If TRUE, type of coefficient (SVC or constant) on x is selected through a BIC (default) or AIC minimization. If FALSE, SVCs are assumed across x. Alternatively, x_sel can be given by column number(s) of x. For example, if x_sel = 2, the coefficient on the second explanatory variable in x is SVC and the other coefficients are constants. The Default is TRUE
x_nvc_sel	If TRUE, type of coefficient (NVC or constant) on x is selected through the BIC (default) or AIC minimization. If FALSE, NVCs are assumed across x. Alternatively, x_nvc_sel can be given by column number(s) of x. For example, if x_nvc_sel = 2, the coefficient on the second explanatory variable in x is NVC and the other coefficients are constants. The Default is TRUE
xconst_nvc_sel	If TRUE, type of coefficient (NVC or constant) on xconst is selected through the BIC (default) or AIC minimization. If FALSE, NVCs are assumed across xconst. Alternatively, xconst_nvc_sel can be given by column number(s) of xconst. For example, if xconst_nvc_sel = 2, the coefficient on the second explanatory variable in xconst is NVC and the other coefficients are constants. The Default is TRUE
nvc_num	Number of basis functions used to model NVC. An intercept and nvc_num natural spline basis functions are used to model each NVC. Default is 5
method	Estimation method. Restricted maximum likelihood method ("reml") and maximum likelihood method ("ml") are available. Default is "reml"
penalty	Penalty to select type of coefficients (SNVC, SVC, NVC, or constant) to stabilize the estimates. The current options are "bic" for the Bayesian information criterion-type penalty ( $N \times \log(K)$ ) and "aic" for the Akaike information criterion (2K) (see Muller et al., 2013). Default is "bic"
maxiter	Maximum number of iterations. Default is 30
tol	The tolerance for matrix inversion. Some errors regarding singular fit can be avoided by reducing the value, but the output can be unstable. Default is 1e-30
covmodel	Type of kernel to model spatial dependence. The currently available options are "exp" for the exponential kernel, "gau" for the Gaussian kernel, and "sph" for the spherical kernel
enum	Number of Moran eigenvectors to be used for spatial process modeling (scalar). Default is 200
bsize	Block/badge size. bsize x bsize elements are iteratively processed during the parallelized computation. Default is 4000
ncores	Number of cores used for the parallel computation. If ncores = NULL, the number of available cores - 2 is detected and used. Default is NULL

**Value**

b_vc	Matrix of estimated SNVC (= SVC + NVC) on x ( $N \times K$ )
bse_vc	Matrix of standard errors for the SNVCs on x ( $N \times k$ )
z_vc	Matrix of z-values for the SNVCs on x ( $N \times K$ )

p_vc	Matrix of p-values for the SNVCs on x (N x K)
B_vc_s	List summarizing estimated SVCs (in SNVC) on x. The four elements are the SVCs (N x K), the standard errors (N x K), z-values (N x K), and p-values (N x K), respectively
B_vc_n	List summarizing estimated NVCs (in SNVC) on x. The four elements are the NVCs (N x K), the standard errors (N x K), z-values (N x K), and p-values (N x K), respectively
c	Matrix with columns for the estimated coefficients on xconst, their standard errors, z-values, and p-values (K_c x 4). Effective if xconst_nvc = FALSE
c_vc	Matrix of estimated NVCs on xconst (N x K_c). Effective if xconst_nvc = TRUE
cse_vc	Matrix of standard errors for the NVCs on xconst (N x K_c). Effective if xconst_nvc = TRUE
cz_vc	Matrix of z-values for the NVCs on xconst (N x K_c). Effective if xconst_nvc = TRUE
cp_vc	Matrix of p-values for the NVCs on xconst (N x K_c). Effective if xconst_nvc = TRUE
s	List of variance parameters in the SNVC (SVC + NVC) on x. The first element is a 2 x K matrix summarizing variance parameters for SVC. The (1, k)-th element is the standard deviation of the k-th SVC, while the (2, k)-th element is the Moran's I value that is scaled to take a value between 0 (no spatial dependence) and 1 (strongest spatial dependence). Based on Griffith (2003), the scaled Moran's I value is interpretable as follows: 0.25-0.50:weak; 0.50-0.70:moderate; 0.70-0.90:strong; 0.90-1.00:marked. The second element of s is the vector of standard deviations of the NVCs
s_c	Vector of standard deviations of the NVCs on xconst
vc	List indicating whether SVC/NVC are removed or not during the BIC/AIC minimization. 1 indicates not removed (replaced with constant) whereas 0 indicates removed
e	Vector whose elements are residual standard error (resid_SE), adjusted conditional R2 (adjR2(cond)), restricted log-likelihood (rlogLik), Akaike information criterion (AIC), and Bayesian information criterion (BIC). When method = "ml", restricted log-likelihood (rlogLik) is replaced with log-likelihood (logLik)
pred	Vector of predicted values (N x 1)
resid	Vector of residuals (N x 1)
other	List of other outputs, which are internally used

### Author(s)

Daisuke Murakami

### References

Muller, S., Scealy, J.L., and Welsh, A.H. (2013) Model selection in linear mixed models. *Statistical Science*, 28 (2), 136-167.

Murakami, D., Yoshida, T., Seya, H., Griffith, D.A., and Yamagata, Y. (2017) A Moran coefficient-based mixed effects approach to investigate spatially varying relationships. *Spatial Statistics*, 19, 68-89.

Murakami, D., and Griffith, D.A. (2019). Spatially varying coefficient modeling for large datasets: Eliminating N from spatial regressions. *Spatial Statistics*, 30, 39-64.

Murakami, D. and Griffith, D.A. (2019) A memory-free spatial additive mixed modeling for big spatial data. *Japan Journal of Statistics and Data Science*. DOI:10.1007/s42081-019-00063-x.

Murakami, D., and Griffith, D.A. (2020) Balancing spatial and non-spatial variations in varying coefficient modeling: a remedy for spurious correlation. *ArXiv*.

## See Also

[resf\\_vc](#), [addlearn\\_local](#)

## Examples

```
require(spdep)
data(boston)
y      <- boston.c[, "CMEDV"]
x      <- boston.c[,c("CRIM", "AGE")]
xconst <- boston.c[,c("ZN", "DIS", "RAD", "NOX", "TAX", "RM", "PTRATIO", "B")]
xgroup <- boston.c[, "TOWN"]
coords <- boston.c[,c("LON", "LAT")]

##### SVC modeling1 #####
##### (SVC on x; Constant coefficients on xconst)
#res    <- besf_vc(y=y,x=x,xconst=xconst,coords=coords, x_sel = FALSE )
#res
#plot_s(res,0) # Spatially varying intercept
#plot_s(res,1) # 1st SVC
#plot_s(res,2) # 2nd SVC
#
##### For large samples (n > 5,000), the following additional learning
##### mitigates an degeneracy/over-smoothing problem in SVCs
#res1    <- addlearn_local(res)
#res1
#plot_s(res1,0) # Spatially varying intercept
#plot_s(res1,1) # 1st SVC
#plot_s(res1,2) # 2nd SVC

##### SVC modeling2 #####
##### (SVC or constant coefficients on x; Constant coefficients on xconst)
#res2    <- besf_vc(y=y,x=x,xconst=xconst,coords=coords )

##### SVC modeling3 #####
##### - Group-level SVC or constant coefficients on x
##### - Constant coefficients on xconst
#res3    <- besf_vc(y=y,x=x,xconst=xconst,coords=coords, s_id=xgroup)

##### SNVC modeling1 #####
```

```
##### - SNVC, SVC, NVC, or constant coefficients on x
##### - Constant coefficients on xconst

#res4  <- besf_vc(y=y,x=x,xconst=xconst,coords=coords, x_nvc =TRUE)

##### SNVC modeling2 #####
##### - SNVC, SVC, NVC, or constant coefficients on x
##### - NVC or Constant coefficients on xconst

#res5  <- besf_vc(y=y,x=x,xconst=xconst,coords=coords, x_nvc =TRUE, xconst_nvc=TRUE)
#plot_s(res5,0)          # Spatially varying intercept
#plot_s(res5,1)          # 1st SNVC (SVC + NVC)
#plot_s(res5,1,btype="svc")# SVC in the 1st SNVC
#plot_n(res5,1,xtype="x") # NVC in the 1st NVC on x
#plot_n(res5,6,xtype="xconst")# NVC in the 6t NVC on xconst
```

---

coef\_marginal

---

*Marginal effects evaluation*


---

## Description

This function evaluates the marginal effects from  $x$  ( $dy/dx$ ) based on the estimation result of [resf](#). This function is for non-Gaussian models transforming  $y$  using [nongauss\\_y](#).

## Usage

```
coef_marginal( mod )
```

## Arguments

mod                      Output from [resf](#)

## Value

b                        Marginal effects from  $x$  ( $dy/dx$ )

## See Also

[resf](#)

---

coef_marginal_vc	<i>Marginal effects evaluation from models with varying coefficients</i>
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---

**Description**

This function evaluates the marginal effects from x (dy/dx) based on the estimation result of [resf\\_vc](#). This funtion is for non-Gaussian models transforming y using [nongauss\\_y](#).

**Usage**

```
coef_marginal_vc( mod )
```

**Arguments**

mod                      Output from [resf\\_vc](#)

**Value**

b_vc	Matrix of the marginal effects of x (dy/dx) (N x K)
B_vc_n	Matrix of the sub-marginal effects of x explained by the spatially varying coefficients (N x K)
B_vc_s	Matrix of the sub-marginal effects explained by the non-spatially varying coefficients (N x K)
c	Matrix of the marginal effects of xconst (N x K_const)
other	List of other outputs, which are internally used

**See Also**

[resf\\_vc](#)

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esf	<i>Spatial regression with eigenvector spatial filtering</i>
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---

**Description**

This function estimates the linear eigenvector spatial filtering (ESF) model. The eigenvectors are selected by a forward stepwise method.

**Usage**

```
esf( y, x = NULL, vif = NULL, meig, fn = "r2" )
```

**Arguments**

<code>y</code>	Vector of explained variables (N x 1)
<code>x</code>	Matrix of explanatory variables (N x K). Default is NULL
<code>vif</code>	Maximum acceptable value of the variance inflation factor (VIF) (scalar). For example, if <code>vif = 10</code> , eigenvectors are selected so that the maximum VIF value among explanatory variables and eigenvectors is equal to or less than 10. Default is NULL
<code>meig</code>	Moran eigenvectors and eigenvalues. Output from <code>meigen</code> or <code>meigen_f</code>
<code>fn</code>	Objective function for the stepwise eigenvector selection. The adjusted R2 ("r2"), AIC ("aic"), or BIC ("bic") are available. Alternatively, all the eigenvectors in <code>meig</code> are used without the stepwise selection if <code>fn = "all"</code> . This is acceptable for large samples (see Murakami and Griffith, 2019). Default is "r2"

**Value**

<code>b</code>	Matrix with columns for the estimated coefficients on <code>x</code> , their standard errors, t-values, and p-values (K x 4)
<code>s</code>	Vector of statistics for the estimated spatial component (2 x 1). The first element is the standard deviation and the second element is the Moran's I value of the estimated spatially dependent component. The Moran's I value is scaled to take a value between 0 (no spatial dependence) and 1 (the maximum possible spatial dependence). Based on Griffith (2003), the scaled Moran's I value is interpretable as follows: 0.25-0.50:weak; 0.50-0.70:moderate; 0.70-0.90:strong; 0.90-1.00:marked
<code>r</code>	Matrix with columns for the estimated coefficients on Moran's eigenvectors, their standard errors, t-values, and p-values (L x 4)
<code>vif</code>	Vector of variance inflation factors of the explanatory variables (N x 1)
<code>e</code>	Vector whose elements are residual standard error ( <code>resid_SE</code> ), adjusted R2 ( <code>adjR2</code> ), log-likelihood ( <code>logLik</code> ), AIC, and BIC
<code>sf</code>	Vector of estimated spatial dependent component ( $E\gamma$ ) (N x 1)
<code>pred</code>	Vector of predicted values (N x 1)
<code>resid</code>	Vector of residuals (N x 1)
<code>other</code>	List of other outputs, which are internally used

**Author(s)**

Daisuke Murakami

**References**

- Griffith, D. A. (2003). Spatial autocorrelation and spatial filtering: gaining understanding through theory and scientific visualization. Springer Science & Business Media.
- Tiefelsdorf, M., and Griffith, D. A. (2007). Semiparametric filtering of spatial autocorrelation: the eigenvector approach. *Environment and Planning A*, 39 (5), 1193-1221.
- Murakami, D. and Griffith, D.A. (2019) Eigenvector spatial filtering for large data sets: fixed and random effects approaches. *Geographical Analysis*, 51 (1), 23-49.

**See Also**[resf](#)**Examples**

```

require(spdep)
data(boston)
y <- boston.c[, "CMEDV" ]
x <- boston.c[,c("CRIM","ZN","INDUS", "CHAS", "NOX","RM", "AGE")]
coords <- boston.c[,c("LON", "LAT")]

#####Distance-based ESF
meig <- meigen(coords=coords)
esfD <- esf(y=y,x=x,meig=meig, vif=5)
esfD

#####Fast approximation
meig_f<- meigen_f(coords=coords)
esfD <- esf(y=y,x=x,meig=meig_f, vif=10, fn="all")
esfD

#####Not run
#####Topoligy-based ESF (it is commonly used in regional science)
#
#cknn <- knearneigh(coordinates(coords), k=4) #4-nearest neighbors
#cmat <- nb2mat(knn2nb(cknn), style="B")
#meig <- meigen(cmat=cmat, threshold=0.25)
#esfT <- esf(y=y,x=x,meig=meig)
#esfT

```

lsem

*Low rank spatial error model (LSEM) estimation***Description**

This function estimates the low rank spatial error model.

**Usage**

```
lsem( y, x, weig, method = "reml" )
```

**Arguments**

y	Vector of explained variables (N x 1)
x	Matrix of explanatory variables (N x K)
weig	eigenvectors and eigenvalues of a spatial weight matrix. Output from <a href="#">weigen</a>
method	Estimation method. Restricted maximum likelihood method ("reml") and maximum likelihood method ("ml") are available. Default is "reml"

**Value**

b	Matrix with columns for the estimated coefficients on x, their standard errors, t-values, and p-values (K x 4)
s	Vector of estimated variance parameters (2 x 1). The first and the second elements denote the estimated rho parameter (sp_lambda) quantifying the scale of spatial dependent process, and the standard error of the process (sp_SE), respectively.
e	Vector whose elements are residual standard error (resid_SE), adjusted conditional R2 (adjR2(cond)), restricted log-likelihood (rlogLik), Akaike information criterion (AIC), and Bayesian information criterion (BIC). When method = "ml", restricted log-likelihood (rlogLik) is replaced with log-likelihood (logLik)
r	Vector of estimated random coefficients on the spatial eigenvectors (L x 1)
pred	Vector of predicted values (N x 1)
resid	Vector of residuals (N x 1)
other	List of other outputs, which are internally used

**Author(s)**

Daisuke Murakami

**References**

Murakami, D., Seya, H. and Griffith, D.A. (2018) Low rank spatial econometric models. Arxiv.

**See Also**

[meigen](#), [meigen\\_f](#)

**Examples**

```
require(spdep)
data(boston)
y <- boston.c[, "CMEDV" ]
x <- boston.c[,c("CRIM", "ZN", "INDUS", "CHAS", "NOX", "RM", "AGE",
                 "DIS", "RAD", "TAX", "PTRATIO", "B", "LSTAT")]
coords<- boston.c[,c("LON", "LAT")]
weig <- weigen( coords )
res  <- lsem(y=y,x=x,weig=weig)
res
```



---

lslm	<i>Low rank spatial lag model (LSLM) estimation</i>
------	---

---

## Description

This function estimates the low rank spatial lag model.

## Usage

```
lslm( y, x, weig, method = "reml", boot = FALSE, iter = 200 )
```

## Arguments

y	Vector of explained variables (N x 1)
x	Matrix of explanatory variables (N x K)
weig	eigenvectors and eigenvalues of a spatial weight matrix. Output from <a href="#">weigen</a>
method	Estimation method. Restricted maximum likelihood method ("reml") and maximum likelihood method ("ml") are available. Default is "reml"
boot	If it is TRUE, confidence intervals for the spatial dependence parameters (s), the mean direct effects (de), and the mean indirect effects (ie), are estimated through a parametric bootstrapping. Default is FALSE
iter	The number of bootstrap replicates. Default is 200

## Value

b	Matrix with columns for the estimated coefficients on x, their standard errors, t-values, and p-values (K x 4)
s	Vector of estimated shrinkage parameters (2 x 1). The first and the second elements denote the estimated rho parameter (sp_rho) quantifying the scale of spatial dependence, and the standard error of the spatial dependent component (sp_SE), respectively. If boot = TRUE, their 95 percent confidence intervals and the resulting p-values are also provided
e	Vector whose elements are residual standard error (resid_SE), adjusted conditional R2 (adjR2(cond)), restricted log-likelihood (rlogLik), Akaike information criterion (AIC), and Bayesian information criterion (BIC). When method = "ml", restricted log-likelihood (rlogLik) is replaced with log-likelihood (logLik)
de	Matrix with columns for the estimated mean direct effects on x. If boot = TRUE, their 95 percent confidence intervals and the resulting p-values are also provided
ie	Matrix with columns for the estimated mean indirect effects on x. If boot = TRUE, their 95 percent confidence intervals and the resulting p-values are also provided
r	Vector of estimated random coefficients on the spatial eigenvectors (L x 1)
pred	Vector of predicted values (N x 1)
resid	Vector of residuals (N x 1)
other	List of other outputs, which are internally used

**Author(s)**

Daisuke Murakami

**References**

Murakami, D., Seya, H. and Griffith, D.A. (2018) Low rank spatial econometric models. Arxiv.

**See Also**[weigen](#), [lsem](#)**Examples**

```

require(spdep)
data(boston)
y <- boston.c[, "CMEDV" ]
x <- boston.c[,c("CRIM", "ZN", "INDUS", "CHAS", "NOX", "RM", "AGE",
                "DIS", "RAD", "TAX", "PTRATIO", "B", "LSTAT")]
coords <- boston.c[,c("LON", "LAT")]
weig <- weigen(coords)
res <- lslm(y=y, x=x, weig=weig)
## res <- lslm(y=y, x=x, weig=weig, boot=TRUE)
res

```

meigen

*Extraction of Moran eigenvectors***Description**

This function extracts spatial and temporal eigenvectors (i.e., basis functions describing spatial and temporal patterns).

**Usage**

```

meigen( coords = NULL, model = "exp", enum = NULL, s_id = NULL,
        threshold = 0, cmat = NULL, coords_z=NULL, enum_z=NULL,
        interact=TRUE, interact_max_dim = 600 )

```

**Arguments**

<code>coords</code>	Matrix of spatial coordinates (N x 2). If <code>cmat</code> is specified, it is ignored
<code>model</code>	Type of kernel to model spatial dependence. The currently available options are "exp" for the exponential kernel, "gau" for the Gaussian kernel, and "sph" for the spherical kernel. Default is "exp"
<code>enum</code>	Optional. The maximum number of spatial eigenvectors to be extracted (scalar)

s_id	Optional. Location/zone ID for modeling inter-group spatial effects. If specified, Moran eigenvectors are extracted by groups. It is useful e.g. for multilevel modeling (s_id is the groups) and panel data modeling (s_id is given by individual location id). Default is NULL
threshold	Optional. Threshold for the eigenvalues. Suppose that lambda_1 is the maximum eigenvalue, this function extracts eigenvectors whose corresponding eigenvalue is equal or greater than (threshold x lambda_1). threshold must be a value between 0 and 1. Default is zero (see Details)
cmat	Optional. A user-specified spatial connectivity matrix (N x N). It must be provided when the user wants to use a spatial connectivity matrix other than the default matrices
coords_z	Optional. One- or two-column matrix whose t-th column represents t-th temporal coordinate (N x 1 or N x 2).
enum_z	Optional. The maximum number of temporal eigenvectors to be extracted (scalar)
interact	Optional. If TRUE, space-time eigenvectors (space x time) are considered in addition to spatial eigenvectors and temporal eigenvectors
interact_max_dim	Optional. The maximum number of the space-time eigenvectors to be extracted (scalar)

## Details

This function extracts spatial eigenvectors from MCM, where  $M = I - 11'/N$  is a centering operator. By default,  $C$  is a  $N \times N$  connectivity matrix whose  $(i, j)$ -th element equals  $\exp(-d(i,j)/h)$ , where  $d(i,j)$  is the spatial Euclidean distance between the sample sites  $i$  and  $j$ .  $h$  is the maximum length of the minimum spanning tree connecting sample sites (see Dray et al., 2006). If  $cmat$  is provided, this function performs the same calculation after  $C$  is replaced with  $cmat$ .

The temporal eigenvectors are extracted in the same way where the spatial distance  $d(i,j)$  is replaced with temporal difference. If two temporal coordinates are given, their eigenvectors are evaluated respectively.

If  $threshold = 0.00$  (default), all the eigenvectors corresponding to positive eigenvalues explaining positive spatial/temporal dependence are extracted.  $threshold = 0.00$  or  $0.25$  are standard assumptions (see Griffith, 2003; Murakami and Griffith, 2015).

## Value

sf	Matrix of the spatial eigenvectors ( $N \times L$ )
ev	Vector of the spatial eigenvalues ( $L \times 1$ ), scaled to have the maximum value of 1
sf_z	List. t-th element is the matrix of the t-th temporal eigenvectors ( $N \times L_t$ )
ev_z	List. t-th element is the vector of the t-th temporal eigenvalues ( $L_t \times 1$ ), scaled to have the maximum value of 1
other	List of other outcomes, which are internally used

## Author(s)

Daisuke Murakami

References

Dray, S., Legendre, P., and Peres-Neto, P.R. (2006) Spatial modelling: a comprehensive framework for principal coordinate analysis of neighbour matrices (PCNM). *Ecological Modelling*, 196 (3), 483-493.

Griffith, D.A. (2003) Spatial autocorrelation and spatial filtering: gaining understanding through theory and scientific visualization. Springer Science & Business Media.

Murakami, D. and Griffith, D.A. (2015) Random effects specifications in eigenvector spatial filtering: a simulation study. *Journal of Geographical Systems*, 17 (4), 311-331.

Murakami, D., Shiota, S., Kajita, S., and Kajita, S. (2024) Fast spatio-temporally varying coefficient modeling with reluctant interaction selection. *ArXiv*.

See Also

[meigen\\_f](#) for fast eigen-decomposition

---

meigen0	<i>Nystrom extension of Moran eigenvectors</i>
---------	--

---

Description

This function estimates Moran eigenvectors at unobserved sites using the Nystrom extension.

Usage

```
meigen0( meig, coords0, coords_z0 = NULL, s_id0 = NULL )
```

Arguments

meig	Moran eigenvectors and eigenvalues. Output from <a href="#">meigen</a> or <a href="#">meigen_f</a>
coords0	Matrix of spatial point coordinates of prediction sites (N_0 x 2)
coords_z0	Optional. One- or two-column matrix whose t-th column represents the t-th temporal coordinate of prediction times (N_0 x 1 or N_0 x 2).
s_id0	Optional. ID specifying groups modeling spatial effects (N_0 x 1). If specified, Moran eigenvectors are extracted by groups. It is useful e.g. for multilevel modeling (s_id is the groups) and panel data modeling (s_id is given by individual location id). Default is NULL

Value

sf	Matrix of the first L eigenvectors at unobserved sites (N_0 x L)
ev	Vector of the first L eigenvalues (L x 1)
sf_z	List. t-th element is the matrix of the t-th temporal eigenvectors (N x L_t)
ev_z	List. t-th element is the vector of the t-th temporal eigenvalues (L_t x 1)
other	List of other outputs, which are internally used

## Author(s)

Daisuke Murakami

## References

Drineas, P. and Mahoney, M.W. (2005) On the Nystrom method for approximating a gram matrix for improved kernel-based learning. *Journal of Machine Learning Research*, 6 (2005), 2153-2175.

## See Also

[meigen](#), [meigen\\_f](#)

---

<i>meigen_f</i>	<i>Fast approximation of Moran eigenvectors</i>
-----------------	---

---

## Description

This function approximates spatial and temporal eigenvectors (i.e., basis functions describing spatial and temporal patterns) computationally efficiently.

## Usage

```
meigen_f( coords, model = "exp", enum = 200, s_id = NULL, threshold = 0,
          coords_z = NULL, enum_z = 200, interact = TRUE,
          interact_max_dim = 600 )
```

## Arguments

<code>coords</code>	Matrix of spatial coordinates (N x 2)
<code>model</code>	Type of kernel to model spatial dependence. The currently available options are "exp" for the exponential kernel, "gau" for the Gaussian kernel, and "sph" for the spherical kernel. Default is "exp"
<code>enum</code>	Number of eigenvectors and eigenvalues to be extracted (scalar). Default is 200
<code>s_id</code>	Optional. Location/zone ID for modeling inter-group spatial effects. If specified, Moran eigenvectors are extracted by groups. It is useful e.g. for multilevel modeling ( <code>s_id</code> is the groups) and panel data modeling ( <code>s_id</code> is given by individual location id). Default is NULL
<code>threshold</code>	Optional. Threshold for the eigenvalues. Suppose that <code>lambda_1</code> is the maximum eigenvalue, this function extracts eigenvectors whose corresponding eigenvalue is equal or greater than ( <code>threshold x lambda_1</code> ). <code>threshold</code> must be a value between 0 and 1. Default is zero
<code>coords_z</code>	Optional. One- or two-column matrix of temporal coordinates (N x 1 or N x 2).
<code>enum_z</code>	Optional. The maximum number of temporal eigenvectors to be extracted (scalar)

interact	Optional. If TRUE, space-time eigenvectors (space x time) are considered in addition to spatial eigenvectors and temporal eigenvectors
interact_max_dim	Optional. The maximum number of the space-time eigenvectors to be extracted (scalar)

### Details

This function extracts approximated spatial eigenvectors from MCM.  $M = I - 11'/N$  is a centering operator, and  $C$  is a spatial connectivity matrix whose  $(i, j)$ -th element is given by  $\exp(-d(i,j)/h)$ , where  $d(i,j)$  is the Euclidean distance between the sample sites  $i$  and  $j$ , and  $h$  is a range parameter given by the maximum length of the minimum spanning tree connecting sample sites (see Dray et al., 2006). Following a simulation result in Murakami and Griffith (2019), this function approximates the 200 eigenvectors corresponding to the 200 largest eigenvalues by default (i.e., `enum = 200`). If `enum` is given by a smaller value like 100, the computation time will be shorter, but with greater approximation error.

The temporal eigenvectors are extracted in the same way where the spatial distance  $d(i,j)$  is replaced with temporal difference. If two temporal coordinates are given, their eigenvectors are evaluated respectively.

### Value

<code>sf</code>	Matrix of the spatial eigenvectors ( $N \times L$ )
<code>ev</code>	Vector of the spatial eigenvalues ( $L \times 1$ ), scaled to have the maximum value of 1
<code>sf_z</code>	List. $t$ -th element is the matrix of the $t$ -th temporal eigenvectors ( $N \times L_t$ )
<code>ev_z</code>	List. $t$ -th element is the vector of the $t$ -th temporal eigenvalues ( $L_t \times 1$ ), scaled to have the maximum value of 1
<code>other</code>	List of other outcomes, which are internally used

### Author(s)

Daisuke Murakami

### References

- Dray, S., Legendre, P., and Peres-Neto, P.R. (2006) Spatial modelling: a comprehensive framework for principal coordinate analysis of neighbour matrices (PCNM). *Ecological Modelling*, 196 (3), 483-493.
- Murakami, D. and Griffith, D.A. (2019) Eigenvector spatial filtering for large data sets: fixed and random effects approaches. *Geographical Analysis*, 51 (1), 23-49.
- Murakami, D., Shirota, S., Kajita, S., and Kajita, S. (2024) Fast spatio-temporally varying coefficient modeling with reluctant interaction selection. *ArXiv*.

### See Also

[meigen](#)

---

nongauss_y	<i>Parameter setup for modeling non-Gaussian continuous data and count data</i>
------------	---

---

## Description

Parameter setup for modeling non-Gaussian continuous data and count data. The SAL transformation (see details) is used to model a wide variety of non-Gaussian data without explicitly assuming data distribution (see Murakami et al., 2021 for further detail). In addition, Box-Cox transformation is used for non-negative continuous variables while another transformation approximating overdispersed Poisson distribution is used for count variables. The output from this function is used as an input of the `resf` and `resf_vc` functions. For further details about its implementation and case study examples, see Murakami (2021).

## Usage

```
nongauss_y( y_type = "continuous", y_nonneg = FALSE, tr_num = 0 )
```

## Arguments

y_type	Type of explained variables y. "continuous" for continuous variables and "count" for count variables
y_nonneg	Effective if y_type = "continuous". TRUE if y cannot take negative value. If y_nonneg = TRUE and tr_num = 0, the Box-Cox transformation is applied to y. If y_nonneg = TRUE and tr_num > 0, the Box-Cox transformation is applied first to roughly Gaussianize y. Then, the SAL transformation is iterated tr_num times to improve the modeling accuracy. Default is FALSE
tr_num	Number of the SAL transformations (SinhArcsinh and Affine, where the use of "L" stems from the "Linear") applied to Gaussianize y. Default is 0

## Details

If `tr_num > 0`, the SAL transformation is iterated `tr_num` times to Gaussianize y. The SAL transformation is defined as  $SAL(y) = a + b \cdot \sinh(c \cdot \operatorname{arcsinh}(y) - d)$  where `a`, `b`, `c`, `d` are parameters. Based on Rios and Tobar (2019), the iteration of the SAL transformation approximates a wide variety of non-Gaussian distributions without explicitly assuming data distribution. The `resf` and `resf_vc` functions return `tr_par`, which is a list whose `k`-th element includes the `a`, `b`, `c`, `d` parameters used for the `k`-th SAL transformation.

In addition, for non-negative y (`y_nonneg = TRUE`), the Box-Cox transformation is applied prior to the iterative SAL transformation. `tr_num` and `y_nonneg` can be selected by comparing the BIC (or AIC) values across models. This compositionally-warped spatial regression approach is detailed in Murakami et al. (2021).

For count data (`y_type = "count"`), an overdispersed Poisson distribution (Gaussian approximation) is assumed. If `tr_num > 0`, the distribution is adjusted to fit the data (y) through the iterative SAL transformations. `y_nonneg` is ignored if `y_type = "count"`.

**Value**

nongauss      List of parameters for modeling non-Gaussian data

**References**

Rios, G. and Tobar, F. (2019) Compositionally-warped Gaussian processes. *Neural Networks*, 118, 235-246.

Murakami, D. (2021) Transformation-based generalized spatial regression using the spmoran package: Case study examples, ArXiv.

Murakami, D., Kajita, M., Kajita, S. and Matsui, T. (2021) Compositionally-warped additive mixed modeling for a wide variety of non-Gaussian data. *Spatial Statistics*, 43, 100520.

Murakami, D., & Matsui, T. (2021). Improved log-Gaussian approximation for over-dispersed Poisson regression: application to spatial analysis of COVID-19. ArXiv, 2104.13588.

**See Also**

[resf](#), [resf\\_vc](#)

**Examples**

```
##### Regression for non-negative data (BC trans.)
ng1 <-nongauss_y( y_nonneg = TRUE )
ng1

##### General non-Gaussian regression for continuous data (two SAL trans.)
ng2 <-nongauss_y( tr_num = 2 )
ng2

##### General non-Gaussian regression for non-negative continuous data
ng3 <-nongauss_y( y_nonneg = TRUE, tr_num = 5 )
ng3

##### Over-dispersed Poisson regression for count data
ng4 <-nongauss_y( y_type = "count" )
ng4

##### A general non-Gaussian regression for count data
ng5 <-nongauss_y( y_type = "count", tr_num = 5 )
ng5

##### Fitting example
require(spdep);require(Matrix)
data(boston)
y <- boston.c[, "CMEDV" ]
x <- boston.c[,c("CRIM","ZN","INDUS", "CHAS", "NOX","RM", "AGE",
                 "DIS", "RAD", "TAX", "PTRATIO", "B", "LSTAT")]
xgroup<- boston.c[, "TOWN"]
coords<- boston.c[,c("LON","LAT")]
meig <- meigen(coords=coords)
res <- resf(y = y, x = x, meig = meig,nongauss=ng2)
```



```

res                      # Estimation results

plot(res$pdf,type="l") # Estimated probability density function
res$skew_kurt           # Skew and kurtosis of the estimated PDF
res$pred_quantile[1:2,]# predicted value by quantile
coef_marginal(res)      # Estimated marginal effects (dy/dx)

```

---

plot_n	<i>Plot non-spatially varying coefficients (NVCs)</i>
--------	---

---

## Description

This function plots non-spatially varying coefficients (NVCs; coefficients varying with respect to explanatory variable value) and their 95 percent confidence intervals

## Usage

```

plot_n( mod, xnum = 1, xtype = "x", cex.lab = 20,
        cex.axis = 15, lwd = 1.5, ylim = NULL, nmax = 20000 )

```

## Arguments

mod	Output from <a href="#">resf</a> , <a href="#">besf</a> , <a href="#">resf_vc</a> , or <a href="#">besf_vc</a> function
xnum	The NVC on the xnum-th explanatory variable is plotted. Default is 1
xtype	Effective for <a href="#">resf_vc</a> and <a href="#">besf_vc</a> . If "x", the num-th NVC in the spatially and non-spatially varying coefficients on x is plotted. If "xconst", the num-th NVC on xconst is plotted. Default is "x"
cex.lab	The size of the x and y axis labels
cex.axis	The size of the tick label numbers
lwd	The width of the line drawing the coefficient estimates
ylim	The limints of the y-axis
nmax	If sample size exceeds nmax, nmax samples are randomly selected and plotted. Default is 20,000

## See Also

[resf](#), [besf](#), [resf\\_vc](#), [besf\\_vc](#)

plot\_qr

*Plot quantile regression coefficients estimated from SF-UQR***Description**

This function plots regression coefficients estimated from the spatial filter unconditional quantile regression (SF-UQR) model.

**Usage**

```
plot_qr( mod, pnum = 1, par = "b", cex.main = 20, cex.lab = 18, cex.axis = 15, lwd = 1.5 )
```

**Arguments**

mod	Output from the <a href="#">resf_qr</a> function
pnum	A number specifying the parameter being plotted. If par = "b", the coefficients on the pnum-th explanatory variable are plotted (intercepts are plotted if pnum = 1). If par = "s" and pnum = 1, the estimated standard errors for the residual spatial process are plotted. If par = "s" and pnum = 2, the Moran's I values of the residual spatial process are plotted. The Moran's I value is scaled to take a value between 0 (no spatial dependence) and 1 (the maximum possible spatial dependence). Based on Griffith (2003), the scaled Moran's I value is interpretable as follows: 0.25-0.50:weak; 0.50-0.70:moderate; 0.70-0.90:strong; 0.90-1.00:marked
par	If it is "b", regression coefficients are plotted. If it is "s", shrinkage (variance) parameters for the residual spatial process are plotted. Default is "b"
cex.main	Graphical parameter specifying the size of the main title
cex.lab	Graphical parameter specifying the size of the x and y axis labels
cex.axis	Graphical parameter specifying the size of the tick label numbers
lwd	Graphical parameters specifying the width of the line drawing the coefficient estimates

**Note**

See [par](#) for the graphical parameters

**See Also**

[resf\\_qr](#)

## Description

This function plots spatially varying coefficients (SVC) and spatio-temporally varying coefficients (STVC) with/without coefficient varying with respect to the value of the explanatory variable (NVC). Namely, the full varying coefficient equals STVC + NVC.

## Usage

```
plot_s( mod, xnum = 0, btype = "all", xtype = "x", pmax = NULL, ncol = 8,
        col = NULL, inv = FALSE, brks = "regular", cex = 1, pch = 20,
        nmax = 20000, coords_z1_lim=NULL, coords_z2_lim = NULL)
```

## Arguments

mod	Output from <a href="#">resf</a> , <a href="#">besf</a> , <a href="#">resf_vc</a> , <a href="#">besf_vc</a> , or <a href="#">addlearn_local</a> function
xnum	For <a href="#">resf_vc</a> , <a href="#">besf_vc</a> , and <a href="#">addlearn_local</a> , xnum-th SVC/STVC is plotted. If num = 0, varying intercept is plotted. For <a href="#">resf</a> and <a href="#">besf</a> , estimated spatially dependent residual process is plotted irrespective of the xnum value. Default is 0
btype	Effective if x_nvc = TRUE in <a href="#">resf_vc</a> and <a href="#">besf_vc</a> . If "all" (default), the estimated varying coefficients (S(T)VC + NVC) are plotted as they are. If "svc", S(T)VC is plotted. If "nvc", NVC is plotted.
xtype	If "x" (default), coefficients on x is plotted. If "xconst", those on xconst is plotted
pmax	The maximum p-value for the varying coefficients to be displayed. For example, if pmax = 0.05, the only coefficients that are statistically significant at the 5 percent level are plotted. If NULL, all the coefficients are plotted. Default is NULL
ncol	Number of colors in the color palette. Default is 8
col	Color palette used for the mapping. If NULL, the blue-pink-yellow color scheme is used. Palettes in the RColorBrewer package are available. Default is NULL
inv	If TRUE, the color palett is inverted. Default is FALSE
brks	If "regular", color is changed at regular intervals. If "quantile", color is changed for each quantile
cex	Size of the dots representing sample sites
pch	A number indicating the symbol to use
nmax	If sample size exceeds nmax, nmax samples are randomly selected and plotted. Default is 20,000
coords_z1_lim	Value range for coords_z[,1] in the <a href="#">meigen/meigen_f</a> function (vector). If is has two elements, the samples whose coords_z[,1] values are in between these values are plotted. If it is a scalar, samples satisfying coords_z[,1]==coords_z1_lim is plotted
coords_z2_lim	Value range for coords_z[,2] (vector).

**See Also**

[resf](#), [besf](#), [resf\\_vc](#), [besf\\_vc](#), [addlearn\\_local](#)

---

predict0

*Spatial and spatio-temporal predictions*

---

**Description**

It is a function for spatial/spatio-temporal prediction using the model estimated from [esf](#), [resf](#), or [resf\\_vc](#) function.

**Usage**

```
predict0( mod, meig0, x0 = NULL, xconst0 = NULL, xgroup0 = NULL, offset0 = NULL,
weight0 = NULL, compute_se=FALSE, compute_quantile = FALSE )
```

**Arguments**

mod	Output from <a href="#">esf</a> <a href="#">resf</a> , or <a href="#">resf_vc</a>
meig0	Moran eigenvectors at prediction sites. Output from <a href="#">meigen0</a>
x0	Matrix of explanatory variables at prediction sites (N_0 x K). Each column of x0 must correspond to those in x in the input model (mod). Default is NULL
xconst0	Effective for <a href="#">resf_vc</a> . Matrix of explanatory variables at prediction sites whose coefficients are assumed constant across space (N_0 x K_const). Each column of xconst0 must correspond to those in xconst in the input model. Default is NULL
xgroup0	Matrix/vector of group IDs at prediction sites that may be integer or name by group (N_0 x K_g). Default is NULL
offset0	Vector of offset variables at prediction sites (N_0 x 1). Effective if y is count (see <a href="#">nongauss_y</a> ). Default is NULL
weight0	Vector of weights for prediction sites (N_0 x 1). Required if compute_se = TRUE or compute_quantile = TRUE, and weight in the input model is not NULL
compute_se	If TRUE, predictive standard error is evaluated. It is currently supported only for continuous variables. If nongauss is specified in the input model (mod), standard error for the transformed y is evaluated. Default is FALSE
compute_quantile	If TRUE, Matrix of the quantiles for the predicted values (N x 15) is evaluated. It is currently supported only for continuous variables. Default is FALSE

**Value**

pred	Matrix with the first column for the predicted values (pred). The second and the third columns are the predicted trend component (xb) and the residual spatial process (sf_residual). If xgroup0 is specified, the fourth column is the predicted group effects (group). If tr_num > 0 or tr_nonneg ==TRUE (i.e., y is transformed) in mod, there is another column of the predicted values in the transformed/normalized scale (pred_trans). In addition, if compute_quantile=TRUE, predictive standard error (pred_se) is evaluated and added as another column
pred_quantile	Effective if compute_quantile = TRUE. Matrix of the quantiles for the predicted values (N x 15). It is useful for evaluating uncertainty in the predictive values
b_vc	Matrix of estimated spatially (spatio-temporally) varying coefficients (S(T)VCs) on x0 (N_0 x K)
bse_vc	Matrix of estimated standard errors for the S(T)VCs (N_0 x K)
t_vc	Matrix of estimated t-values for the S(T)VCs (N_0 x K)
p_vc	Matrix of estimated p-values for the S(T)VCs (N_0 x K)
c_vc	Matrix of estimated non-spatially varying coefficients (NVCs) on x0 (N x K). Effective if nvc =TRUE in <a href="#">resf</a>
cse_vc	Matrix of standard errors for the NVCs on x0 (N x K).Effective if nvc =TRUE in <a href="#">resf</a>
ct_vc	Matrix of t-values for the NVCs on x0 (N x K). Effective if nvc =TRUE in <a href="#">resf</a>
cp_vc	Matrix of p-values for the NVCs on x0 (N x K). Effective if nvc =TRUE in <a href="#">resf</a>

**See Also**[meigen0](#)**Examples**

```

require(spdep)
data(boston)
samp    <- sample( dim( boston.c )[ 1 ], 300)

d      <- boston.c[ samp, ]    ## Data at observed sites
y      <- d[, "CMEDV"]
x      <- d[,c("ZN", "LSTAT")]
xconst <- d[,c("CRIM", "NOX", "AGE", "DIS", "RAD", "TAX", "PTRATIO", "B", "RM")]
coords <- d[,c("LON", "LAT")]

d0     <- boston.c[-samp, ]    ## Data at unobserved sites
y0     <- d0[, "CMEDV"]
x0     <- d0[,c("ZN", "LSTAT")]
xconst0 <- d0[,c("CRIM", "NOX", "AGE", "DIS", "RAD", "TAX", "PTRATIO", "B", "RM")]
coords0 <- d0[,c("LON", "LAT")]

meig    <- meigen( coords = coords )
meig0   <- meigen0( meig = meig, coords0 = coords0 )

```

```
##### Spatial prediction #####
#### model with residual spatial dependence
mod    <- resf(y=y, x=x, meig=meig)
pred0  <- predict0( mod = mod, x0 = x0, meig0 = meig0 )
pred0$pred[1:5,] # Predicted values

#### model with spatially varying coefficients (SVCs)
mod    <- resf_vc(y=y, x=x, xconst=xconst, meig=meig )
pred0  <- predict0( mod = mod, x0 = x0, xconst0=xconst0, meig0 = meig0 )
pred0$pred[1:5,] # Predicted values
pred0$b_vc[1:5,] # SVCs
pred0$bse_vc[1:5,]# standard errors of the SVCs
pred0$t_vc[1:5,] # t-values of the SNVCs
pred0$p_vc[1:5,] # p-values of the SNVCs

plot(y0,pred0$pred[,1]);abline(0,1)
```

resf

*spatial and spatio-temporal regression models*

## Description

This model estimates regression coefficients, coefficients varying depending on  $x$  (non-spatially varying coefficients; NVC), and group effects, considering residual spatial/spatio-temporal dependence. The random-effects eigenvector spatial filtering, which is an approximate Gaussian process approach, is used for modeling the residual dependence. If `nonugauss` is specified, non-Gaussian explained variables are Gaussianized using a compositional warping function (see [nongauss\\_y](#)). This argument allows the `resf` function to be applied to non-Gaussian explained variables, including count data.

## Usage

```
resf( y, x = NULL, xgroup = NULL, weight = NULL, offset = NULL,
      nvc = FALSE, nvc_sel = TRUE, nvc_num = 5, meig,
      method = "reml", penalty = "bic", nongauss = NULL )
```

## Arguments

<code>y</code>	Vector of explained variables (N x 1)
<code>x</code>	Matrix of explanatory variables (N x K). Default is NULL
<code>xgroup</code>	Matrix of group IDs. The IDs may be group numbers or group names (N x K_g). Default is NULL
<code>weight</code>	Vector of weights for samples (N x 1). If non-NULL, the adjusted R-squared value is evaluated for weighted explained variables. Default is NULL
<code>offset</code>	Vector of offset variables (N x 1). Available if <code>y</code> is count ( <code>y_type = "count"</code> is specified in the <a href="#">nongauss_y</a> function). Default is NULL

nvc	If TRUE, a non-linear function of x (NVC; a spline function) is used as a varying coefficient. If FALSE, constant coefficients are assumed. Default is FALSE
nvc_sel	If TRUE, type of each coefficient (NVC or constant) is selected through a BIC minimization. If FALSE, NVCs are assumed across x. Alternatively, nvc_sel can be given by column number(s) of x. For example, if nvc_sel = 2, the coefficient on the second explanatory variable is NVC and the other coefficients are constants. Default is TRUE
nvc_num	Number of natural spline basis functions to be used to model NVC. Default is 5
meig	Moran eigenvectors and eigenvalues. Output from <a href="#">meigen</a> or <a href="#">meigen_f</a>
method	Estimation method. Restricted maximum likelihood method ("reml") and maximum likelihood method ("ml") are available. Default is "reml"
penalty	Penalty to select type of coefficients (NVC or constant) to stabilize the estimates. The current options are "bic" for the Bayesian information criterion-type penalty ( $N \times \log(K)$ ) and "aic" for the Akaike information criterion (2K). Default is "bic"
nongauss	Parameter setup for modeling non-Gaussian continuous data or count data. Output from <a href="#">nongauss_y</a>

## Details

For modeling non-Gaussian data including count data, see [nongauss\\_y](#).

## Value

b	Matrix with columns for the estimated constant coefficients on x, their standard errors, t-values, and p-values ( $K \times 4$ )
b_g	List of $K_g$ matrices with columns for the estimated group effects, their standard errors, and t-values
c_vc	Matrix of estimated NVCs on x ( $N \times K$ ). Effective if nvc = TRUE
cse_vc	Matrix of standard errors for the NVCs on x ( $N \times K$ ). Effective if nvc = TRUE
ct_vc	Matrix of t-values for the NVCs on x ( $N \times K$ ). Effective if nvc = TRUE
cp_vc	Matrix of p-values for the NVCs on x ( $N \times K$ ). Effective if nvc = TRUE
s	Vector of estimated variance parameters ( $2 \times 1$ ). The first and the second elements are the standard deviation and the Moran's I value of the estimated spatially (and temporally) dependent process, respectively. The Moran's I value is scaled to take a value between 0 (no spatial dependence) and 1 (the maximum possible spatial dependence). Based on Griffith (2003), the scaled Moran's I value is interpretable as follows: 0.25-0.50:weak; 0.50-0.70:moderate; 0.70-0.90:strong; 0.90-1.00:marked
s_c	Vector of standard deviations of the NVCs on xconst
s_g	Vector of estimated standard deviations of the group effects
e	Error statistics. When y_type="continuous", it includes residual standard error (resid_SE), adjusted conditional R2 (adjR2(cond)), restricted log-likelihood (rlogLik), Akaike information criterion (AIC), and Bayesian information criterion (BIC). rlogLik is replaced with log-likelihood (logLik) if method = "ml".

	resid_SE is replaced with the residual standard error for the transformed y (resid_SE_trans) if nongauss is specified. When y_type="count", the error statistics contains root mean squared error (RMSE), Gaussian likelihood approximating the model, AIC and BIC based on the likelihood, and the proportion of the null deviance explained by the model (deviance explained (%)). deviance explained, which is also used in the mgcv package, corresponds to the adjusted R2 in case of the linear regression
vc	List indicating whether NVC are removed or not during the BIC minimization. 1 indicates not removed whereas 0 indicates removed
r	Vector of estimated random coefficients on the Moran's eigenvectors (L x 1)
sf	Vector of estimated spatial dependent component (N x 1)
pred	Matrix of predicted values for y (pred) and their standard errors (pred_se) (N x 2). If y is transformed by specifying <a href="#">nongauss_y</a> , the predicted values in the transformed/normalized scale are added as another column named pred_trans
pred_quantile	Matrix of the quantiles for the predicted values (N x 15). It is useful to evaluate uncertainty in the predictive value
tr_par	List of the parameter estimates for the tr_num SAL transformations. The k-th element of the list includes the four parameters for the k-th SAL transformation (see <a href="#">nongauss_y</a> )
tr_bpar	The estimated parameter in the Box-Cox transformation
tr_y	Vector of the transformed explained variables
resid	Vector of residuals (N x 1)
pdf	Matrix whose first column consists of evenly spaced values within the value range of y and the second column consists of the estimated value of the probability density function for y if y_type in <a href="#">nongauss_y</a> is "continuous" and probability mass function (PMF) if y_type = "count". If offset is specified (and y_type = "count"), the PMF given median offset value is evaluated
skew_kurt	Skewness and kurtosis of the estimated probability density/mass function of y
other	List of other outputs, which are internally used

### Author(s)

Daisuke Murakami

### References

- Murakami, D. and Griffith, D.A. (2015) Random effects specifications in eigenvector spatial filtering: a simulation study. *Journal of Geographical Systems*, 17 (4), 311-331.
- Murakami, D., and Griffith, D.A. (2020) Balancing spatial and non-spatial variations in varying coefficient modeling: a remedy for spurious correlation. *Geographical Analysis*, DOI: 10.1111/gean.12310.
- Murakami, D., Kajita, M., Kajita, S. and Matsui, T. (2021) Compositionally-warped additive mixed modeling for a wide variety of non-Gaussian data. *Spatial Statistics*, 43, 100520.
- Murakami, D., Shirota, S., Kajita, S., and Kajita, S. (2024) Fast spatio-temporally varying coefficient modeling with reluctant interaction selection. *ArXiv*.



**See Also**

[meigen](#), [meigen\\_f](#), [coef\\_marginal](#), [besf](#)

**Examples**

```
#####
##### Spatial regression modeling #####
#####
require(spdep);require(Matrix)
data(boston)
y <- boston.c[, "CMEDV" ]
x <- boston.c[,c("CRIM","ZN","INDUS", "CHAS", "NOX","RM", "AGE",
                 "DIS", "RAD", "TAX", "PTRATIO", "B", "LSTAT")]
xgroup<- boston.c[, "TOWN"]
coords<- boston.c[,c("LON","LAT")]
meig <- meigen(coords=coords)
# meig<- meigen_f(coords=coords) ## for large samples

#####
##### Gaussian regression #####
res <- resf(y = y, x = x, meig = meig)
res
plot_s(res) ## spatially dependent component (intercept)

#### Group-wise random intercepts
#res2 <- resf(y = y, x = x, meig = meig, xgroup = xgroup)

#### Group-level spatial dependence (s_id) + random intercepts (xgroup)
#meig_g<- meigen(coords=coords, s_id = xgroup)
#res3 <- resf(y = y, x = x, meig = meig_g, xgroup = xgroup)

#### Coefficients varying depending on x
#res4 <- resf(y = y, x = x, meig = meig, nvc = TRUE)
#res4

#plot_s(res4) # spatially dependent component (intercept)
#plot_s(res4,5) # spatial plot of the 5-th NVC
#plot_s(res4,6) # spatial plot of the 6-th NVC
#plot_s(res4,13)# spatial plot of the 13-th NVC

#plot_n(res4,5) # 1D plot of the 5-th NVC
#plot_n(res4,6) # 1D plot of the 6-th NVC
#plot_n(res4,13)# 1D plot of the 13-th NVC

#####
##### Non-Gaussian regression #####

#### Model for non-Gaussian continuous data
# - Probability distribution is estimated from data
```

```

#ng5    <- nongauss_y( tr_num = 2 )# 2 SAL transformations to Gaussianize y
#res5    <- resf(y = y, x = x, meig = meig, nongauss = ng5)
#res5      ## tr_num may be selected by comparing BIC

#plot(res5$pdf,type="l") # Estimated probability density function
#res5$skew_kurt          # Skew and kurtosis of the estimated PDF
#res5$pred_quantile[1:2,]# predicted value by quantile
#coef_marginal(res5)     # Estimated marginal effects (dy/dx)

#### Model for non-Gaussian and non-negative continuous data
# - Probability distribution is estimated from data

#ng6    <- nongauss_y( tr_num = 2, y_nonneg = TRUE )
#res6    <- resf(y = y, x = x, meig = meig, nongauss = ng6 )
#coef_marginal(res6)

#### Overdispersed Poisson model for count data
# - y: count data

#ng7    <- nongauss_y( y_type = "count" )
#res7    <- resf(y = y, x = x, meig = meig, nongauss = ng7 )

#### General model for count data
# - y: count data
# - Probability distribution is estimated from data

#ng8    <- nongauss_y( y_type = "count", tr_num = 2 )
#res8    <- resf(y = y, x = x, meig = meig, nongauss = ng8 )

#####
##### STVC modeling #####
#####
# See \url{https://github.com/dmuraka/spmoran}

#require(spData)
#data(house)
#dat0    <- st_as_sf(house)
#dat     <- data.frame(st_coordinates(dat0), dat0)
#y       <- log(dat[, "price"])
#x       <- dat[,c("lotsize", "TLA", "rooms", "beds")]

#byear   <- house$yrbuilt
#syear   <- as.numeric(as.character(house$syear))#factor -> numeric
#coords_z<- cbind(byear,syear)
#meig    <- meigen_f(coords=coords, coords_z=cbind(byear,syear),interact=TRUE)
#res9    <- resf(y=y,x=x,meig=meig )
#res9

```

---

resf\_qr

*Spatial filter unconditional quantile regression***Description**

This function estimates the spatial filter unconditional quantile regression (SF-UQR) model.

**Usage**

```
resf_qr( y, x = NULL, meig, tau = NULL, boot = TRUE,
        iter = 200, parallel=FALSE, ncores=NULL )
```

**Arguments**

y	Vector of explained variables (N x 1)
x	Matrix of explanatory variables (N x K). Default is NULL
meig	Moran eigenvectors and eigenvalues. Output from <a href="#">meigen</a> or <a href="#">meigen_f</a>
tau	The quantile(s) to be modeled. It must be a number (or a vector of numbers) strictly between 0 and 1. By default, tau = c(0.1, 0.2, ..., 0.9)
boot	If it is TRUE, confidence intervals of regression coefficients are estimated by a semiparametric bootstrapping. Default is TRUE
iter	The number of bootstrap replications. Default is 200
parallel	If TRUE, the bootstrapping for estimating confidence intervals is parallelized. Default is FALSE
ncores	Number of cores used for the parallel computation. If ncores=NULL, which is the default, the number of available cores - 2 is detected and used

**Value**

b	Matrix of estimated regression coefficients (K x Q), where Q is the number of quantiles (i.e., the length of tau)
r	Matrix of estimated random coefficients on Moran eigenvectors (L x Q)
s	Vector of estimated variance parameters (2 x 1). The first and the second elements denote the standard deviation and the Moran's I value of the estimated spatially dependent component, respectively. The Moran's I value is scaled to take a value between 0 (no spatial dependence) and 1 (the maximum possible spatial dependence). Based on Griffith (2003), the scaled Moran's I value is interpretable as follows: 0.25-0.50:weak; 0.50-0.70:moderate; 0.70-0.90:strong; 0.90-1.00:marked
e	Vector whose elements are residual standard error (resid_SE) and adjusted quasi conditional R2 (quasi_adjR2(cond))
B	Q matrices (K x 4) summarizing bootstrapped estimates for the regression coefficients. Columns of these matrices consist of the estimated coefficients, the lower and upper bounds for the 95 percent confidential intervals, and p-values. It is returned if boot = TRUE

S	Q matrices (2 x 3) summarizing bootstrapped estimates for the variance parameters. Columns of these matrices consist of the estimated parameters, the lower and upper bounds for the 95 percent confidential intervals. It is returned if boot = TRUE
B0	List of Q matrices (K x iter) summarizing bootstrapped coefficients. The q-th matrix consists of the coefficients on the q-th quantile. Effective if boot = TRUE
S0	List of Q matrices (2 x iter) summarizing bootstrapped variance parameters. The q-th matrix consists of the parameters on the q-th quantile. Effective if boot = TRUE

### Author(s)

Daisuke Murakami

### References

Murakami, D. and Seya, H. (2017) Spatially filtered unconditional quantile regression. ArXiv.

### See Also

[plot\\_qr](#)

### Examples

```
require(spdep)
data(boston)
y <- boston.c[, "CMEDV" ]
x <- boston.c[,c("CRIM", "ZN", "INDUS", "CHAS", "NOX", "RM", "AGE",
                 "DIS", "RAD", "TAX", "PTRATIO", "B", "LSTAT")]
coords <- boston.c[,c("LON", "LAT")]
meig <- meigen(coords=coords)
res <- resf_qr(y=y,x=x,meig=meig, boot=FALSE)
res
plot_qr(res,1)      # Intercept
plot_qr(res,2)      # Coefficient on CRIM
plot_qr(res,1,"s")  # spcomp_SE
plot_qr(res,2,"s")  # spcomp_Moran.I/max(Moran.I)

###Not run
#res <- resf_qr(y=y,x=x,meig=meig, boot=TRUE)
#res
#plot_qr(res,1)      # Intercept + 95 percent confidence interval (CI)
#plot_qr(res,2)      # Coefficient on CRIM + 95 percent CI
#plot_qr(res,1,"s")  # spcomp_SE + 95 percent CI
#plot_qr(res,2,"s")  # spcomp_Moran.I/max(Moran.I) + 95 percent CI
```

---

resf_vc	<i>spatial and spatio-temporal regression models with varying coefficients</i>
---------	--

---

## Description

This function estimates spatially varying coefficients (SVC) or spatio-temporally varying coefficients (STVC), group effects, considering residual spatial/spatio-temporal dependence. A non-linear function of  $x$  (NVC) can be added on each SVC/STVC mainly to stabilize the estimation (see Murakami and Griffith, 2020). Approximate Gaussian processes based on Moran eigenvectors are used for modeling the spatio-temporal processes. Type of coefficients (constant or varying) is selected through a BIC minimization. If `nongauss` is specified, non-Gaussian explained variables are Gaussianized using a compositional warping function (see [nongauss\\_y](#)). This argument allows the `resf` function to be applied to non-Gaussian explained variables, including count data.

Note that, for very large samples, this function can overlook small-scale spatial variations. [addlearn\\_local](#) applies an model aggregation/averaging technique to address this problem.

## Usage

```
resf_vc(y, x, xconst = NULL, xgroup = NULL, weight = NULL, offset = NULL,
        x_nvc = FALSE, xconst_nvc = FALSE, x_sel = TRUE, x_nvc_sel = TRUE,
        xconst_nvc_sel = TRUE,
        nvc_num = 5, meig, method = "reml", penalty = "bic", nongauss = NULL,
        miniter=NULL, maxiter = 30, tol = 1e-30 )
```

## Arguments

<code>y</code>	Vector of explained variables ( $N \times 1$ )
<code>x</code>	Matrix of explanatory variables assuming SVC/STVC ( $N \times K$ )
<code>xconst</code>	Matrix of explanatory variables assuming constant coefficients ( $N \times K_c$ ). Default is NULL
<code>xgroup</code>	Matrix of group IDs for modeling group-wise random effects. The IDs may be group numbers or group names ( $N \times K_g$ ). Default is NULL
<code>weight</code>	Vector of weights for samples ( $N \times 1$ ). If non-NULL, the adjusted R-squared value is evaluated for weighted explained variables. Default is NULL
<code>offset</code>	Vector of offset variables ( $N \times 1$ ). Available if <code>y</code> is count ( <code>y_type = "count"</code> is specified in the <a href="#">nongauss_y</a> function). Default is NULL
<code>x_nvc</code>	If TRUE, a non-linear function of $x$ (NVC) is added on each varying coefficient on $x$ to stabilize the estimate. Default is FALSE
<code>xconst_nvc</code>	If TRUE, NVCs is added on each constant coefficient on <code>xconst</code> model estimate non-linear influence from <code>xconst</code>
<code>x_sel</code>	If TRUE, type of coefficient on $x$ (STVC, SVC, or constant) is selected through a BIC minimization. If FALSE, S(T)VCs are assumed across $x$ . Alternatively, <code>x_sel</code> can be given by column number(s) of $x$ . For example, if <code>x_sel = 2</code> , the coefficient on the second explanatory variable is S(T)VC and the other coefficients are constants. The Default is TRUE

x_nvc_sel	If TRUE, with/without NVC on x is selected. If FALSE, NVCs are assumed across x. Alternatively, x_nvc_sel can be given by column number(s) of x. For example, if x_nvc_sel = 2, the coefficient on the second explanatory variable is NVC and the other coefficients are constants. The Default is TRUE
xconst_nvc_sel	If TRUE, with/without NVC on xconst is selected. If FALSE, NVCs are assumed across xconst. Alternatively, xconst_nvc_sel can be given by column number(s) of xconst. For example, if xconst_nvc_sel = 2, the coefficient on xconst[,2] becomes constant + NVC while the other coefficients become constants. The Default is TRUE
nvc_num	Number of natural spline basis functions to be used in NVC. Default is 5
meig	Moran eigenvectors and eigenvalues. Output from <a href="#">meigen</a> or <a href="#">meigen_f</a>
method	Estimation method. Restricted maximum likelihood method ("reml") and maximum likelihood method ("ml") are available. Default is "reml"
penalty	Penalty for model estimation and selection. "bic" for the Bayesian information criterion-type penalty ( $N \times \log(K)$ ) and "aic" for the Akaike information criterion ( $2K$ ). Default is "bic"
nongauss	Parameter setup for modeling non-Gaussian continuous and count data. Output from <a href="#">nongauss_y</a>
miniter	Minimum number of iterations. Default is NULL
maxiter	Maximum number of iterations. Default is 30
tol	The tolerance for matrix inversion. Some errors regarding singular fit can be avoided by reducing the value, but the output can be unstable. Default is $1e-30$

## Details

For modeling non-Gaussian data including count data, see [nongauss\\_y](#).

## Value

b_vc	Matrix of estimated spatially/spatio-temporally varying coefficients (S(T)VC + NVC) on x ( $N \times K$ )
bse_vc	Matrix of standard errors for the varying coefficients on x ( $N \times k$ )
t_vc	Matrix of t-values for the coefficients on x ( $N \times K$ )
p_vc	Matrix of p-values for the coefficients on x ( $N \times K$ )
B_vc_s	List of the estimated S(T)VCs in b_vc (= S(T)VC + NVC). The elements are the S(T)VCs ( $N \times K$ ), the standard errors ( $N \times K$ ), t-values ( $N \times K$ ), and p-values ( $N \times K$ ), respectively
B_vc_n	List of the estimated NVCs in b_vc (= S(T)VC + NVC). The elements are the NVCs ( $N \times K$ ), the standard errors ( $N \times K$ ), t-values ( $N \times K$ ), and p-values ( $N \times K$ ), respectively
c	Matrix with columns for the estimated coefficients on xconst, their standard errors, t-values, and p-values ( $K_c \times 4$ ). Effective if xconst_nvc = FALSE
c_vc	Matrix of estimated NVCs on xconst ( $N \times K_c$ ). Effective if xconst_nvc = TRUE

cse_vc	Matrix of standard errors for the NVCs on xconst (N x k_c). Effective if xconst_nvc = TRUE
ct_vc	Matrix of t-values for the NVCs on xconst (N x K_c). Effective if xconst_nvc = TRUE
cp_vc	Matrix of p-values for the NVCs on xconst (N x K_c). Effective if xconst_nvc = TRUE
b_g	List of K_g matrices with columns for the estimated group effects, their standard errors, and t-values
s	List of the variance parameters for the varying coefficient on x. The first element is a 2 x K matrix summarizing variance parameters for S(T)VC. The (1, k)-th element is the standard deviation of the k-th SVC, while the (2, k)-th element is the Moran's I value that is scaled to take a value between 0 (no spatial dependence) and 1 (strongest spatial dependence). Based on Griffith (2003), the scaled Moran'I value is interpretable as follows: 0.25-0.50:weak; 0.50-0.70:moderate; 0.70-0.90:strong; 0.90-1.00:marked. The second element of s is the vector of standard deviations of the NVCs
s_c	Vector of standard deviations of the NVCs on xconst
s_g	Vector of standard deviations of the group effects
vc	List indicating whether S(T)VC/NVC are removed or not during the BIC minimization. 1 indicates not removed (replaced with constant) whereas 0 indicates removed
e	Error statistics. When y_type="continuous", it includes residual standard error (resid_SE), adjusted conditional R2 (adjR2(cond)), restricted log-likelihood (rlogLik), Akaike information criterion (AIC), and Bayesian information criterion (BIC). rlogLik is replaced with log-likelihood (logLik) if method = "ml". resid_SE is replaced with the residual standard error for the transformed y (resid_SE_trans) if nongauss is specified. When y_type="count", the error statistics includes root mean squared error (RMSE), Gaussian likelihood approximating the model, AIC and BIC based on the likelihood, and the proportion of the null deviance explained by the model (deviance explained (%)). deviance explained, which is also used in the mgcv package, corresponds to the adjusted R2 in case of the linear regression
pred	Matrix of predicted values for y (pred) and their standard errors (pred_se) (N x 2). If y is transformed by specifying <a href="#">nongauss_y</a> , the predicted values in the transformed/normalized scale are added as another column named pred_trans
pred_quantile	Matrix of the quantiles for the predicted values (N x 15). It is useful to evaluate uncertainty in the predictive value
tr_par	List of the parameter estimates for the tr_num SAL transformations. The k-th element of the list includes the four parameters for the k-th SAL transformation (see <a href="#">nongauss_y</a> )
tr_bpar	The estimated parameter in the Box-Cox transformation
tr_y	Vector of the transformed explained variables
resid	Vector of residuals (N x 1)

pdf	Matrix whose first column consists of evenly spaced values within the value range of y and the second column consists of the estimated value of the probability density function for y if y_type in <a href="#">nongauss_y</a> is "continuous" and probability mass function if y_type = "count". If offset is specified (and y_type = "count"), the PMF given median offset value is evaluated
skew_kurt	Skewness and kurtosis of the estimated probability density/mass function of y
other	List of other outputs, which are internally used

### Author(s)

Daisuke Murakami

### References

- Murakami, D., Yoshida, T., Seya, H., Griffith, D.A., and Yamagata, Y. (2017) A Moran coefficient-based mixed effects approach to investigate spatially varying relationships. *Spatial Statistics*, 19, 68-89.
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- Griffith, D. A. (2003) *Spatial autocorrelation and spatial filtering: gaining understanding through theory and scientific visualization*. Springer Science & Business Media.

### See Also

[meigen](#), [meigen\\_f](#), [coef\\_marginal](#), [besf\\_vc](#), [addlearn\\_local](#)

### Examples

```
#####
##### SVC modeling #####
#####
require(spdep)
data(boston)
y      <- boston.c[, "CMEDV"]
x      <- boston.c[,c("CRIM", "AGE")]
xconst <- boston.c[,c("ZN", "DIS", "RAD", "NOX", "TAX", "RM", "PTRATIO", "B")]
xgroup <- boston.c[, "TOWN"]
coords <- boston.c[,c("LON", "LAT")]
meig   <- meigen(coords=coords)
# meig <- meigen_f(coords=coords) ## for large samples

#####
##### Gaussian regression with SVC #####

res    <- resf_vc(y=y, x=x, xconst=xconst, meig=meig )
```



```

res
plot_s(res,0) # Spatially varying intercept
plot_s(res,1) # 1st SVC (Not shown because the SVC is estimated constant)
plot_s(res,2) # 2nd SVC

#### For large samples (e.g., n > 5,000), the following
#### additional learning often improves the modeling accuracy
# res_adj<- addlearn_local(res)
# res_adj
# plot_s(res_adj,0)
# plot_s(res_adj,1)
# plot_s(res_adj,2)

#### Group-level SVC (s_id) + random intercepts (xgroup)
# meig_g <- meigen(coords, s_id=xgroup)
# res2 <- resf_vc(y=y,x=x,xconst=xconst,meig=meig_g,xgroup=xgroup)

#####
##### Gaussian regression with SVC + NVC #####

# res3 <- resf_vc(y=y,x=x,xconst=xconst,meig=meig, x_nvc =TRUE)
# plot_s(res3,0) # Spatially varying intercept
# plot_s(res3,1) # Spatial plot of the varying coefficient (SVC + NVC) on x[,1]
# plot_s(res3,1,btype="svc")# Spatial plot of SVC in the coefficient
# plot_s(res3,1,btype="nvc")# Spatial plot of NVC in the coefficient
# plot_n(res3,1) # 1D plot of the NVC

#####
##### Non-Gaussian regression with SVC #####

#### Model for non-Gaussian continuous data
# - Probability distribution is estimated from data

# ng4 <- nongauss_y( tr_num = 2 )# 2 SAL transformations to Gaussianize y
# res4 <- resf_vc(y=y,x=x,xconst=xconst,meig=meig, nongauss = ng4 )
# res4 # tr_num may be selected by comparing BIC

# coef_marginal_vc(res4) # marginal effects from x (dy/dx)
# plot(res4$pdf,type="l") # Estimated probability density function
# res4$skew_kurt # Skew and kurtosis of the estimated PDF
# res4$pred_quantile[1:2,]# predicted value by quantile

#### Model for non-Gaussian and non-negative continuous data
# - Probability distribution is estimated from data

## 2 SAL trans. + 1 Box-Cox trans. to Gaussianize y
# ng5 <- nongauss_y( tr_num = 2, y_nonneg = TRUE )
# res5 <- resf_vc(y=y,x=x,xconst=xconst,meig=meig, nongauss = ng5 )
# coef_marginal_vc(res5)

```

```
#### Overdispersed Poisson model for count data
# - y: count data

#ng6   <- nongauss_y( y_type = "count" )
#res6   <- resf_vc(y=y,x=x,xconst=xconst,meig=meig, nongauss = ng6 )

#### General model for count data
# - y: count data
# - Probability distribution is estimated from data

#ng7   <- nongauss_y( y_type = "count", tr_num = 2 )
#res7   <- resf_vc(y=y,x=x,xconst=xconst,meig=meig, nongauss = ng7 )

#####
##### STVC modeling #####
#####
# See \url{https://github.com/dmuraka/spmoran}

#require(spData)
#data(house)
#dat0   <- st_as_sf(house)
#dat    <- data.frame(st_coordinates(dat0), dat0)
#y      <- log(dat[, "price"])
#x      <- dat[, c("lotsize", "TLA")]
#xconst <- dat[, c("rooms", "beds")]

#byear  <- house$yrbuilt
#syear  <- as.numeric(as.character(house$syear))#factor -> numeric
#coords_z<- cbind(byear,syear)
#meig   <- meigen_f(coords=coords, coords_z=cbind(byear,syear),interact=TRUE)
#res8   <- resf_vc(y=y,x=x,xconst=xconst,meig=meig )
#res8

## Varying intercept for byear <=1950 and syear==1998
#plot_s(res8,0, coords_z1_lim=c(-Inf, 1950),coords_z2_lim=1998)

## 1st STVCs which are significant at the 5 percent level, for byear <= 1950
#plot_s(res8,1, coords_z1_lim=c(-Inf, 1950), pmax=0.05)

## 2nd STVC for byear >= 1951
#plot_s(res8,2, coords_z1_lim=c(1951,Inf))
```

---

weigen

---

*Extract eigenvectors from a spatial weight matrix*


---

## Description

This function extracts eigenvectors and eigenvalues from a spatial weight matrix.

**Usage**

```
weigen( x = NULL, type = "knn", k = 4, threshold = 0.25, enum = NULL )
```

**Arguments**

x	Matrix of spatial point coordinates (N x 2), sf polygon object (N spatial units), or an user-specified spatial weight matrix (N x N) (see Details)
type	Type of spatial weights. The currently available options are "knn" for the k-nearest neighbor-based weights, and "tri" for the Delaunay triangulation-based weights. If sf polygons are provided for x, type is ignored, and the rook-type neighborhood matrix is created
k	Number of nearest neighbors. It is used if type ="knn"
threshold	Threshold for the eigenvalues (scalar). Suppose that $\lambda_1$ is the maximum eigenvalue. Then, this function extracts eigenvectors whose corresponding eigenvalues are equal or greater than $[\text{threshold} \times \lambda_1]$ . It must be a value between 0 and 1. Default is 0.25 (see Details)
enum	Optional. The maximum acceptable number of eigenvectors to be used for spatial modeling (scalar)

**Details**

If user-specified spatial weight matrix is provided for x, this function returns the eigen-pairs of the matrix. Otherwise, if sf polygon object is provided to x, the rook-type neighborhood matrix is created using this polygon, and eigen-decomposed. Otherwise, if point coordinates are provided to x, a spatial weight matrix is created according to type, and eigen-decomposed.

By default, the ARPACK routine is implemented for fast eigen-decomposition.

threshold = 0.25 (default) is a standard setting for topology-based ESF (see Tiefelsdorf and Griffith, 2007) while threshold = 0.00 is a usual setting for distance-based ESF.

**Value**

sf	Matrix of the first L eigenvectors (N x L)
ev	Vector of the first L eigenvalues (L x 1)
other	List of other outcomes, which are internally used

**Author(s)**

Daisuke Murakami

**References**

- Tiefelsdorf, M. and Griffith, D.A. (2007) Semiparametric filtering of spatial autocorrelation: the eigenvector approach. *Environment and Planning A*, 39 (5), 1193-1221.
- Murakami, D. and Griffith, D.A. (2018) Low rank spatial econometric models. Arxiv, 1810.02956.

**See Also**

[meigen](#), [meigen\\_f](#)

**Examples**

```
require(spdep)
data(boston)
if (require("spData", quietly=TRUE)) {
##### Rook adjacency-based W
poly      <- st_read(system.file("shapes/boston_tracts.gpkg", package="spData")[1])
weig1     <- weigen( poly )

##### knn-based W
coords    <- boston.c[,c("LON", "LAT")]
weig2     <- weigen( coords, type = "knn" )

##### Delaunay triangulation-based W
coords    <- boston.c[,c("LON", "LAT")]
weig3     <- weigen( coords, type = "tri" )

##### User-specified W
dmat      <- as.matrix(dist(coords))
cmat      <- exp(-dmat)
diag(cmat)<- 0
weig4     <- weigen( cmat, threshold = 0 )
}
```

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